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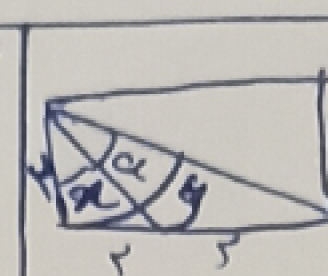
محمد شیرانی 11B

۱- $\frac{6\times\sqrt{3}\times\sin\alpha}{2}=\frac{9}{2}\Rightarrow\sin\alpha=\frac{\sqrt{3}}{2}\begin{cases}\alpha=60^\circ\\ \alpha=120^\circ\end{cases}$

$\frac{\max\alpha}{\min\alpha}=2$ ✓ (۲)

۲- (۲)



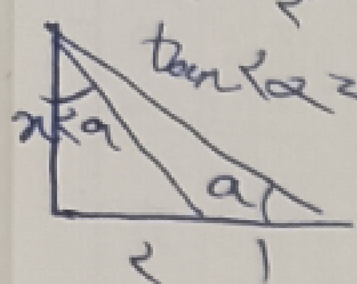
$\tan\alpha=\tan(y-x)$

$\tan y=\frac{4}{2}=2$

$\tan x=\frac{2}{2}=1$

$\frac{2-1}{1+2}=\frac{1}{3}\to\boxed{\cot\alpha=3}$ ✓

۳- (۲)



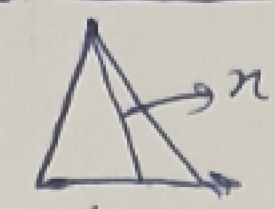
$\tan\alpha=\frac{x}{3}$

$\tan 2\alpha=\frac{2}{x}=\frac{2\tan\alpha}{1-\tan^2\alpha}$

$\frac{6x}{9-x^2}=\frac{2}{x}\to x=\frac{3}{2}$

$\Rightarrow\tan\alpha=\frac{3}{2}\times\frac{1}{3}=\frac{1}{2}\to\boxed{\cot\alpha=2}$ ✓

۴- طبق قضیه استوارت داریم: (۲)



$x^2=\frac{16+80}{6}-5\Rightarrow x=\sqrt{11}$

قضیه cos: $11+1-2\sqrt{11}\cos\alpha=16\to\cos\alpha=-\frac{2}{\sqrt{11}}$

$\to 1+\tan^2\alpha=\frac{1}{\cos^2\alpha}\Rightarrow\tan^2\alpha=\frac{7}{4}\to\tan\alpha=\frac{-\sqrt{7}}{2}$ ✓

۵- (۲)

$\sin^2\alpha+\underbrace{\sin^2\alpha+\cos^2\alpha}_{1}=\frac{4}{3}\to\begin{cases}\sin^2\alpha=\frac{1}{3}\\ \cos^2\alpha=\frac{2}{3}\end{cases}$

$1+\tan^2\alpha=\frac{1}{\cos^2\alpha}\Rightarrow\frac{3}{2}-1=\boxed{\frac{1}{2}=\tan^2\alpha}$ ✓

۶- (۲)

$\sin^4\alpha+4\cos^2\alpha=\sin^4\alpha-4\sin^2\alpha+4$

$\cos^4\alpha+4\sin^2\alpha=\cos^4\alpha-4\cos^2\alpha+4$

$1+\cos^2\alpha=2-\sin^2\alpha$

$1+\sin^2\alpha=2-\cos^2\alpha$

$\frac{(\sin^2\alpha-2)^2}{2-\sin^2\alpha}-\frac{(\cos^2\alpha-2)^2}{2-\cos^2\alpha}$

$\to 2-\sin^2\alpha-2+\cos^2\alpha\to\boxed{\cos^2\alpha-\sin^2\alpha=\cos2\alpha}$ ✓

۷- (۱)

$\sin(\frac{9\pi}{2}+\alpha)=\cos\alpha$

$\cos(\frac{7\pi}{2}-\alpha)=-\sin\alpha$

$-\tan(\alpha-\frac{3\pi}{2})=\tan(\frac{3\pi}{2}-\alpha)=\cot\alpha$

$\boxed{\cos\alpha\cdot(-\sin\alpha+\cot\alpha)}$ ✓ $=-\sin\alpha\cos\alpha+\frac{\cos\alpha}{\sin\alpha}$

$\Rightarrow\frac{-\sin^2\alpha\cos\alpha+\cos\alpha}{\sin\alpha}=\frac{\cos\alpha(1-\sin^2)}{\sin\alpha}$

$\Rightarrow\frac{\cos\alpha\cdot\cos^2\alpha}{\sin\alpha}=\frac{\cos^2\alpha}{\sin\alpha}$

$\to\cot\alpha\cdot\cos^2\alpha$

۸- می‌دانیم: (۲)

$\cos\frac{\pi}{12}=\frac{\sqrt{6}+\sqrt{2}}{4}$

$\sin\frac{\pi}{12}=\frac{\sqrt{6}-\sqrt{2}}{4}$

$\sqrt{2}(\sin x-\cos x)=\frac{\sqrt{6}-\sqrt{2}-\sqrt{6}-\sqrt{2}}{4}=\frac{-2\sqrt{2}}{4}=\frac{-\sqrt{2}}{2}$

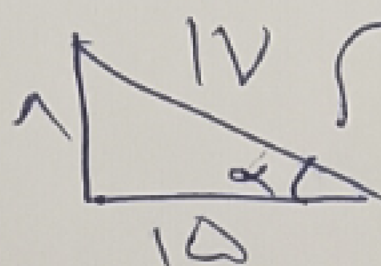
$\sqrt{2}(\frac{-\sqrt{2}}{2})=-1$

$2\cos4x=2\cos\frac{\pi}{3}=2\times\frac{1}{2}=\frac{2}{2}$

$\frac{3}{2}-1=\frac{1}{2}$ ✓

۹- (۲)

$\tan\alpha=\frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}=\frac{\frac{1}{2}}{\frac{15}{16}}=\frac{8}{15}\to$



$\sin\alpha=\frac{8}{17}$

$\cos\alpha=\frac{15}{17}$

$\tan\alpha=\frac{8}{15}$

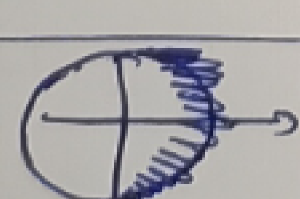
$\cot\alpha=\frac{15}{8}$

$\frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}}=\frac{\frac{16}{255}}{\frac{-7}{17}}=\frac{-16}{105}$ ✓

۱۰- (۲)

$0<\frac{\cot\alpha}{\sin\alpha}\Rightarrow 0<\frac{\cos\alpha}{\sin^2\alpha}$

$\cos\alpha\to(+)$, $(+)\leftarrow\sin^2\alpha$



ربع ۱ و ۴ $\cos\alpha>0$

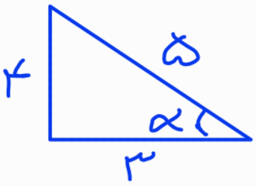
$2\sin\alpha<2\sin\alpha\cos\alpha\xrightarrow{\text{if }\sin\alpha>0}1<\cos\alpha\to$ غ ق ق ✗

$\xrightarrow{\text{if }\sin\alpha<0}1>\cos\alpha\to$ ق ق ✓

ناحیه ۴ ✓

finish

۷_



$\sin\alpha = -\frac{۴}{۵}$ $\cos\alpha = -\frac{۳}{۵}$

$\cos\alpha(-\sin\alpha) + \cot\alpha = \left(-\frac{۳}{۵}\right)\left(\frac{۴}{۵}\right) + \frac{۳}{۴} = ۰٫۲۷$