۱۹,۷۵ — سمیر علای

۲۷

۱-

$S = \frac{1}{2} \times 4 \times \sqrt{3} \times \sin\alpha = ۴,۵$

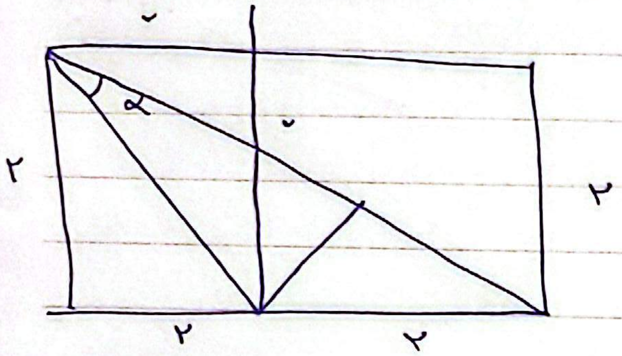
$\sin\alpha = \frac{\sqrt{3}}{2} \Rightarrow \alpha_1 = 60$

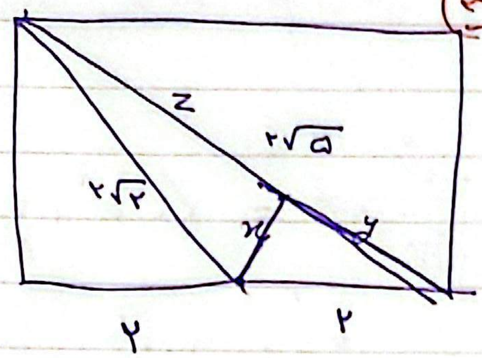
$\alpha_2 = 120$

$\frac{\alpha_2}{\alpha_1} = \frac{120}{60} = 2$

(۲)

۲- (۳) (۲)





$x^2 + (2\sqrt{5} - y)^2 = 8$

$x^2 + y^2 = 4$

$x^2 = 4 - y^2$

$4 - y^2 + y^2 + 20 - 4\sqrt{5}\,y = 8$

$24 - 4\sqrt{5}\,y = 8$

$16 = 4\sqrt{5}\,y$

$4 = \sqrt{5}\,y$

$y = \frac{4}{\sqrt{5}}$

$\left(\frac{4}{\sqrt{5}}\right)^2 + x^2 = 4$

$\frac{16}{5} + x^2 = 4$

$x^2 = \frac{4}{5}$ $\boxed{x = \frac{2}{\sqrt{5}}}$ $\frac{2}{\sqrt{5}}$

$\left(\frac{2}{\sqrt{5}}\right)^2 + (z)^2 = 8$

$\frac{4}{5} + z^2 = \frac{40}{5}$

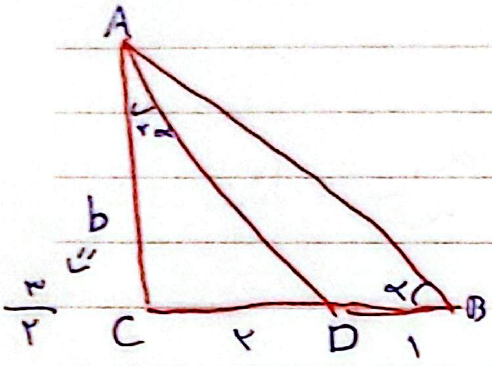
$z^2 = \frac{36}{5}$

$\boxed{z = \frac{6}{\sqrt{5}}}$

$\boxed{\cot\alpha = \frac{z}{x} = \frac{\frac{6}{\sqrt{5}}}{\frac{2}{\sqrt{5}}} = 3}$

۳-

۲



$\cot\alpha = \frac{a}{b} = \frac{۳}{b}$

$\tan(۲\alpha) = \frac{۲}{b} = \frac{۲\tan\alpha}{۱-\tan^۲\alpha} = \frac{۲\times\frac{b}{۳}}{۱-\frac{b^۲}{۹}}$

$\frac{۹-b^۲}{\cancel{۹}۳}\times\cancel{۲} = \cancel{۲}\times b^۲\times\frac{۱}{\cancel{۳}}$

$b^۲ = \frac{۹-b^۲}{۳} \Rightarrow ۳b^۲ = ۹-b^۲$

$۴b^۲ = ۹$

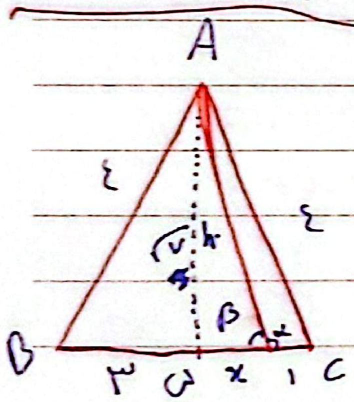
$b^۲ = \frac{۹}{۴} \quad b = \frac{۳}{۲}$

$\cot\alpha = \frac{a}{b} = \frac{۳}{\frac{۳}{۲}} = ۲$

۴-

$\frac{\sqrt{۷}}{۲}$

۱٫۷۵



$\alpha+\beta = ۱۸۰$

$|\tan\alpha| = |\tan\beta|$

$\frac{۵+۱}{۲} = ۳ \quad ۳-۱ = ۲$

$\left.\begin{array}{l}(۵-x)^۲+h^۲ = ۱۶\\ (۱+x)^۲+h^۲ = ۱۶\end{array}\right] \Rightarrow x = ۲$

$h^۲ = ۱۶-۹$

$h = \sqrt{۷}$

$|\tan\beta| = |\tan\alpha| = \frac{\sqrt{۷}}{۲} = \frac{\sqrt{۷}}{۲}$

$\tan\alpha = -\frac{\sqrt{۷}}{۲}$

$\sin^2 + \underbrace{\sin^2 + \cos^2}_{1} = \frac{4}{3}$ — ۵

$\sin^2 = \frac{1}{3} \Rightarrow |\sin| = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ ۲

$\sin^2 = \frac{1}{3} \quad \sin^2 + \cos^2 = 1 \Rightarrow \cos^2 = \frac{2}{3}$

$\tan^2\alpha = \frac{\sin^2}{\cos^2} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$ ✓ $\quad \cos = \sqrt{\frac{2}{3}}$

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۶. $\sin^4\alpha = (\sin^2\alpha)^2 = (1-\cos^2)^2 = 1 + \cos^4 - 2\cos^2 \Rightarrow$

۲

$$\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} = \frac{1+\cos^4\alpha + 2\cos^2\alpha}{1+\cos^2\alpha} = \frac{(1+\cos^2\alpha)^2}{1+\cos^2\alpha} = 1+\cos^2\alpha \quad (1)$$

$$\frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} = \frac{1+\sin^4\alpha - 2\sin^2\alpha + 4\sin^2\alpha}{1+\sin^2} = \frac{(1+\sin^2)^2}{1+\sin^2} =$$

$1+\sin^2\alpha \quad (2)$

$(1) - (2) = 1 + \cos^2\alpha - 1 - \sin^2\alpha = \cos^2\alpha - \sin^2\alpha = \cos(2\alpha)$

$\cos(2\alpha)$ ✓

$\sin\left(\frac{9\pi}{2}\right) = \left(\frac{\pi}{2}\right)$

$\left(\frac{7\pi}{2}\right) : \frac{3\pi}{2}$

$\sin\left(\alpha + \frac{\pi}{2}\right) = \quad < 0$

$\left(\sin\alpha \cos\frac{\pi}{2} + \cos\alpha \sin\frac{\pi}{2}\right) \Rightarrow$ ①

$\cos\left(\frac{3\pi}{2} - \alpha\right) =$

②

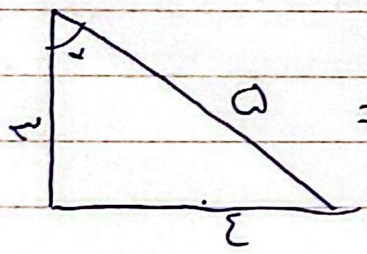
$\sin\left(\alpha + \frac{\pi}{2}\right) = \cos(\alpha)$

$\cos\left(\frac{7\pi}{2} - \alpha\right) = -\sin\alpha$

$\tan\left(\alpha - \frac{3\pi}{2}\right) = -\cot(\alpha) \Rightarrow$

$-\cot\alpha$

$\cos(\alpha) - \sin(\alpha) + \cot(\alpha) = -\sin\cos + \cot(\alpha)$



$\Rightarrow \begin{cases} \sin = -\frac{4}{5} \\ \cos = -\frac{3}{5} \end{cases}$

$-\sin_\alpha \cos_\alpha =$

$\cancel{\frac{4}{5}}$

$-\sin\cos + \cot_\alpha = \frac{-12}{25} + \frac{3}{4} = \frac{27}{100}$ ✓

$x = \frac{\pi}{۱۲}$

۸

$۳\cos\frac{۴\pi}{۱۲} + \sqrt{۲}\left(\sin\frac{\pi}{۱۲} - \cos\frac{\pi}{۱۲}\right)$

۲

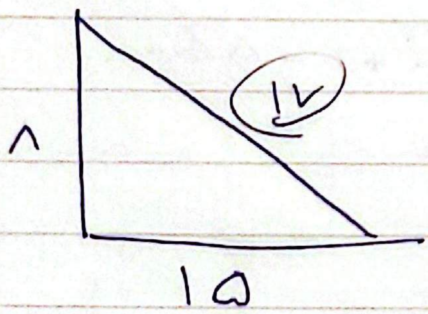
$۳ \times \frac{۱}{۲} + \sqrt{۲} \times \sqrt{۲}\left(\sin\left(\frac{\pi}{۱۲} - \frac{\pi}{۴}\right)\right) =$

$\frac{۳}{۲} + ۲ \times \sin\left(-\frac{\pi}{۶}\right) = \frac{۳}{۲} + \left(۲ \times -\frac{۱}{۲}\right) = \frac{۳}{۲} - \frac{۲}{۲} = \frac{۱}{۲}$ ✓

۹-

$\tan(۲\alpha) = \frac{۲\tan\alpha}{۱ - \tan^۲\alpha} \Rightarrow \tan(\alpha) = \frac{\frac{۱}{\cancel{۳}}}{\frac{۱۵}{\cancel{۲۴}\,۸}} = \frac{۸}{۱۵} \Rightarrow$

۲



$\left.\begin{aligned} \sin\alpha &= \frac{۸}{۱۷} \\ \cos &= \frac{۱۵}{۱۷} \end{aligned}\right] \Rightarrow$

$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{۸}{۱۵} - \frac{۸}{۱۷}}{\frac{۸}{۱۷} - \frac{۱۵}{۱۷}} = \frac{\frac{۱۶}{\cancel{۲۵۵}\ ۱۵}}{\frac{-۷}{\cancel{۱۷}}} = -\frac{۱۶}{۱۰۵}$ ✓

۶۴+۲۲۵= ۲۸۹

۱۰-

$$0 < \frac{\cot\alpha}{\sin\alpha} \Rightarrow 0 < \frac{\cos\alpha}{\sin^2\alpha} \Rightarrow \cos\alpha > 0$$

همواره مثبت

ناحیه ۱ و ۴

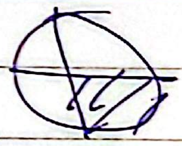
۲

$$2\sin\alpha < 2\sin\alpha\cos\alpha$$

بین ۰ و ۱ پس

$\sin\alpha < 0$ زیرا اگر مثبت
بود بعد از ضرب شدن در یک عدد
بین ۰ و ۱ کوچکتر می‌شود
پس $\sin\alpha < 0$ است

$$\left.\begin{array}{l}\sin\alpha < 0\\ \cos\alpha > 0\end{array}\right\} \Rightarrow$$



ناحیه چهارم