یازدهم تجربی ۲ — تکلیف ۲۷ — محمدامین جعفری

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$S = \frac{1}{2} \times 6 \times \sqrt{3} \times \sin\alpha = 4/5 \Rightarrow 3\sqrt{3}\sin\alpha = \frac{9}{2} \Rightarrow \sin\alpha = \frac{\sqrt{3}}{2}$

$\xrightarrow{\alpha < 180^\circ} \alpha = \begin{cases} 60^\circ \\ 120^\circ \end{cases} \Rightarrow \frac{120^\circ}{60^\circ} = 2$

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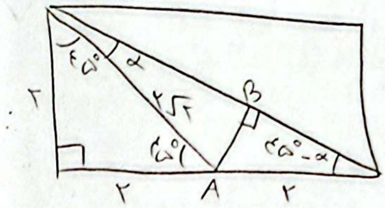

$\sin\alpha = \frac{AB}{2\sqrt{2}}$

$\sin(45-\alpha) = \frac{AB}{2} = \sin 45\cos\alpha - \sin\alpha\cos 45$

$= \frac{\sqrt{2}}{2}\cos\alpha - \frac{\sqrt{2}}{2}\sin\alpha = \frac{\sqrt{2}}{2}(\cos\alpha - \sin\alpha)$

$\sin(45-\alpha) = \sqrt{2}\sin\alpha \Rightarrow \frac{\sqrt{2}}{2}(\cos\alpha - \sin\alpha) = \sqrt{2}\sin\alpha \Rightarrow \cos\alpha - \sin\alpha = 2\sin\alpha$

$\Rightarrow \cos\alpha = 3\sin\alpha \rightarrow \cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{3\sin\alpha}{\sin\alpha} = 3$

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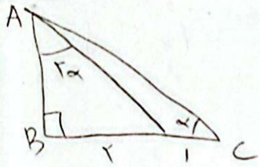

$\left.\begin{array}{l} \tan(2\alpha) = \frac{3}{AB} \\ \tan\alpha = \frac{AB}{3} \end{array}\right\} \Rightarrow \tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} \Rightarrow \frac{3}{AB} = \frac{\frac{2AB}{3}}{1-\frac{AB^2}{9}}$

$\Rightarrow \frac{3}{AB} = \frac{\frac{2AB}{3}}{\frac{9-AB^2}{9}} \Rightarrow \frac{2AB^2}{3} = \frac{18-2AB^2}{9} \Rightarrow 6AB^2 = 18 - 2AB^2 \Rightarrow 8AB^2 = 18$

$\Rightarrow AB^2 = \frac{18}{8} = \frac{9}{4} \Rightarrow AB = \frac{3}{2} \Rightarrow \cot\alpha = \frac{1}{\frac{AB}{3}} = \frac{3}{AB} = \frac{3}{\frac{3}{2}} = \frac{6}{3} = 2$

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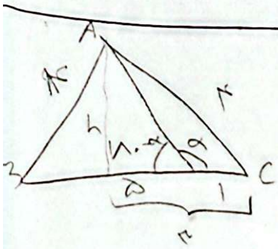

$h = \sqrt{4^2 - 3^2} = \sqrt{7} \Rightarrow \tan(180^\circ - \alpha) = \frac{\sqrt{7}}{3}$

$\tan\alpha = -\tan(180 - \alpha) = -\frac{\sqrt{7}}{3}$

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$2\sin^2 x + \cos^2 x = \frac{4}{3} \xrightarrow{\sin^2 x + \cos^2 x = 1} 1 + \sin^2 x = \frac{4}{3} \Rightarrow \sin^2 x = \frac{1}{3}$

$2\sin^2 x + \cos^2 x = \frac{4}{3} \xrightarrow{2(\sin^2 x + \cos^2 x) = 2\sin^2 x + 2\cos^2 x = 2} 2\sin^2 x + 2\cos^2 x - \cos^2 x = 2 - \cos^2 x = \frac{4}{3}$

$\Rightarrow \cos^2 x = \frac{2}{3}$

$\Rightarrow \tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$

$$= \frac{\sin^4\alpha + 4(1-\sin^2\alpha)}{1+(1-\sin^2\alpha)} - \frac{(1-\sin^2\alpha)^2 + 4\sin^2\alpha}{1+\sin^2\alpha}$$ 6

$$= \frac{\sin^4\alpha - 4\sin^2\alpha + 4}{2-\sin^2\alpha} - \frac{\sin^4\alpha - 2\sin^2\alpha + 1 + 4\sin^2\alpha}{1+\sin^2\alpha} = 2-\sin^2\alpha - (1+\sin^2\alpha)$$

$$= 1 - 2\sin^2\alpha = \cos^2\alpha - \sin^2\alpha = \cos 2\alpha$$

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$$= \sin\left(\frac{\pi}{2}+\alpha\right)\cos\left(\frac{3\pi}{2}-\alpha\right) - \tan\left(\alpha+\frac{\pi}{2}\right) = (\cos\alpha \times -\sin\alpha) + \cot\alpha$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{4}{3} \Rightarrow \sin\alpha = \frac{4}{3}\cos\alpha \Rightarrow \sin^2\alpha + \cos^2\alpha = \frac{16}{9}\cos^2\alpha + \frac{9}{9}\cos^2\alpha = \frac{25}{9}\cos^2\alpha = 1$$

$$\Rightarrow \cos^2\alpha = \frac{9}{25} \Rightarrow \cos\alpha \xrightarrow{\text{ربع سوم}} -\frac{3}{5} \Rightarrow \sin\alpha = -\frac{4}{5} \Rightarrow \text{عبارت} = \left(-\frac{3}{5}\times\frac{4}{5}\right) + \frac{3}{4} = \frac{-12}{25} + \frac{3}{4}$$

$$= \frac{-48+75}{100} = \frac{27}{100}$$

8

$$\text{عبارت} = \left(2\cos\frac{\pi}{3} + \sqrt{2}\left(\sqrt{2}\sin\left(\alpha - \frac{\pi}{4}\right)\right)\right) = \frac{3}{2} + 2\sin\left(\frac{\pi}{12} - \frac{4\pi}{12}\right)$$

$$= \frac{3}{2} + 2\sin\left(-\frac{\pi}{4}\right) = \frac{3}{2} + 2\left(-\frac{1}{2}\right) = \frac{1}{2}$$

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$$\tan(\alpha) = \tan\left(2\times\frac{\alpha}{2}\right) = \frac{2\tan\left(\frac{\alpha}{2}\right)}{1-\tan^2\left(\frac{\alpha}{2}\right)} = \frac{2\left(\frac{1}{4}\right)}{1-\frac{1}{16}} = \frac{\frac{1}{2}}{\frac{15}{16}} = \frac{8}{15} \Rightarrow \sin\alpha = \frac{8}{15}\cos\alpha$$

$$\Rightarrow \sin^2\alpha + \cos^2\alpha = \frac{64}{225}\cos^2\alpha + \cos^2\alpha = \frac{289}{225}\cos^2\alpha = 1 \Rightarrow \cos^2\alpha = \frac{225}{289} \Rightarrow \cos\alpha = \frac{15}{17}$$

$$\Rightarrow \sin\alpha = \frac{8}{17}$$

$$\Rightarrow \text{عبارت} = \frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{\frac{136-120}{255}}{\frac{-7}{17}} = \frac{\frac{16}{255}}{\frac{-7}{17}} = \frac{\frac{16}{15}}{-7} = \frac{-16}{105}$$

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$$2\sin\alpha < \sin 2\alpha \Rightarrow 2\sin\alpha\cos\alpha > 2\sin\alpha \Rightarrow 2\sin\alpha\cos\alpha - 2\sin\alpha > 0$$

$$\frac{\cot\alpha}{\sin\alpha} > 0 \Rightarrow \frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha} > 0 \Rightarrow \frac{\cos\alpha}{\sin^2\alpha} > 0 \Rightarrow \cos\alpha > 0$$

$$\Rightarrow 2\sin\alpha(\cos\alpha - 1) > 0 \xrightarrow{\cos\alpha \in (0,1]} 2\sin\alpha < 0 \Rightarrow \sin\alpha < 0$$

$\Rightarrow$ ربع چهارم