نام و نام خانوادگی: پارسا ایردانی

تکالیف شماره ۲۷ کلاس یازدهم پسر B

۱- $S_{ABC} = \frac{1}{2}\sin\alpha \times 6\times\sqrt{3} = 6 \Rightarrow \sin\alpha = \frac{6}{3\sqrt{3}} = \frac{\sqrt{3}}{2}$

$\frac{\text{بزرگتر}}{\text{کوچکتر}} = \frac{120}{60} = 2$ ✓

$\sin 60^\circ = \frac{\sqrt{3}}{2}$ ، $\sin 120^\circ = \frac{\sqrt{3}}{2}$

۲- $\cot\alpha = 3 \Rightarrow \cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{\frac{3}{\sqrt{10}}}{\frac{1}{\sqrt{10}}} = 3$ ✓

$\frac{b}{\sin\beta} = \frac{a}{\sin\alpha} \Rightarrow \frac{2\sqrt{2}}{\frac{1}{\sqrt{5}}} = \frac{2}{\sin\alpha} \Rightarrow \sin\alpha = \frac{1}{\sqrt{10}}$

$\sin\beta = \frac{2}{2\sqrt{5}} = \frac{1}{\sqrt{5}}$

$\sin^2\alpha + \cos^2\alpha = 1 \Rightarrow \frac{1}{10} + \cos^2\alpha = 1 \Rightarrow \cos^2\alpha = \frac{9}{10} \Rightarrow \cos\alpha = \frac{3}{\sqrt{10}}$

۳- $\tan 2\alpha = \frac{2}{x}$ ، $\tan\alpha = \frac{x}{2}$

$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} \Rightarrow \frac{2}{x} = \frac{x}{1-\frac{x^2}{9}} \Rightarrow 1-\frac{x^2}{9} = \frac{x^2}{3} \Rightarrow 1 = \frac{4x^2}{9}$

$\Rightarrow x^2 = \frac{9}{4} \Rightarrow x = \frac{3}{2}$

$\tan\alpha = \frac{\frac{3}{2}}{3} = \frac{1}{2} \Longrightarrow \cot\alpha = 2$ ✓

۴- مثلث ABC متساوی‌الساقین $\Rightarrow AB = AC$

$AH = \sqrt{AC^2 - CH^2} = \sqrt{4^2 - 3^2} = \sqrt{7}$

$\tan\alpha = \tan(180 - \beta) = -\tan\beta = -\frac{\sqrt{7}}{2}$ ✓

۵- $2\sin^2 x + \cos^2 x = \frac{4}{3} \Rightarrow 1 + \sin^2 x = \frac{4}{3} \Rightarrow \sin^2 x = \frac{1}{3}$ ، $\cos^2 x = \frac{2}{3}$

$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$ ✓

۶- $\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} \rightarrow \frac{\sin^4\alpha + 3 - 3\sin^2\alpha}{1+1-\sin^2\alpha} - \frac{\cos^4\alpha + 3 - 3\cos^2\alpha}{1+1-\cos^2\alpha} \Rightarrow \frac{(\sin^2\alpha - 2)^2}{2-\sin^2\alpha} - \frac{(2-\cos^2\alpha)^2}{2-\cos^2\alpha}$

$\Rightarrow 2 - \sin^2\alpha - (2 - \cos^2\alpha) \Rightarrow 2 - \sin^2\alpha - 2 + \cos^2\alpha \Rightarrow \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha$

۷- $\sin\left(\frac{9\pi}{2} + \alpha\right) = \sin\left(\frac{\pi}{2} + \alpha\right) = +\cos\alpha$

$\cos\left(\frac{7\pi}{2} - \alpha\right) = \cos\left(\frac{3\pi}{2} - \alpha\right) = -\sin\alpha$

$\tan\left(\alpha - \frac{3\pi}{2}\right) = -\cot\alpha$

فیثاغورس $\Rightarrow 5 = x = 5 \Rightarrow \cos\alpha = -\frac{3}{5}$ ، $\sin\alpha = \frac{4}{5}$ ، $\cot\alpha = \frac{3}{4}$

$(\cos\alpha)(-\sin\alpha) - (-\cot\alpha) = \left(-\frac{3}{5}\right)\left(\frac{4}{5}\right) + \frac{3}{4} = \frac{-12}{25} + \frac{3}{4} = \frac{27}{100}$ ✓

۸- $x = \frac{\pi}{12}$ $\quad 4\cos\frac{\pi}{3} + \sqrt{2}\sin\frac{\pi}{12} - \sqrt{2}\cos\frac{\pi}{12} = \frac{4}{2} + \sqrt{2}\left(\sin\frac{\pi}{12} - \cos\frac{\pi}{12}\right) = \frac{3}{2} + \sqrt{2} \times \frac{-1}{\sqrt{2}} = \frac{1}{2}$ ✓

$A = \sin\frac{\pi}{12} - \cos\frac{\pi}{12}$ $\xrightarrow{\sin < \cos}$ $A < 0$ $\quad A^2 = \left(\sin\frac{\pi}{12} - \cos\frac{\pi}{12}\right)^2 = \sin^2\frac{\pi}{12} + \cos^2\frac{\pi}{12} - 2\sin\frac{\pi}{12}\cos\frac{\pi}{12}$

$\Rightarrow A^2 = 1 - \sin\frac{\pi}{6} = 1 - \frac{1}{2} = 1 \xrightarrow[A<0]{\sqrt{\ }} A = -\frac{1}{\sqrt{2}}$

۹- $\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} = \frac{2\left(\frac{1}{4}\right)}{1-\frac{1}{16}} = \frac{5}{15}$

فیثاغورس $\xrightarrow{}$ $BC = 17 \Rightarrow \begin{cases} \sin\alpha = \frac{8}{17} \\ \cos\alpha = \frac{15}{17} \end{cases}$

$\Rightarrow \frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{5}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{7}} = \frac{14}{105}$

$2\sin\alpha < \sin 2\alpha \Rightarrow 2\sin\alpha < 2\sin\alpha\cos\alpha \Rightarrow \underbrace{2\sin\alpha - 2\sin\cos\alpha < 0}_{2\sin\alpha(1-\cos\alpha)<0} \Rightarrow \underline{\sin\alpha < 0}$ همواره نامنفی

$\frac{\cot\alpha}{\sin\alpha} > 0 \Rightarrow \underline{\cot\alpha < 0}$ ربع ۴

-۱۵