۱۹,۲۵

نام و نام خانوادگی ....سهیل بهرامی........ پاسخنامه تشریحی تکلیف شماره ...۲۸... کلاس ....یازدهم سبز B........

۱ (۲)

$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \Rightarrow \cancel{\frac{1}{|\cos\alpha|}} - \tan\alpha = \cancel{\frac{1}{|\cos\alpha|}} - \frac{\sin\alpha}{|\cos\alpha|} \Rightarrow \frac{\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{|\cos\alpha|} \Rightarrow \cos\alpha > 0$

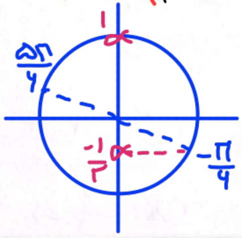
$\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} = \frac{\cos\alpha}{\sqrt{\sin^2\alpha}} = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow \sin\alpha > 0$

α در ناحیه اول قرار دارد

۲ (۱,۷۵)

$-\frac{\pi}{12} < x < \frac{5\pi}{12} \Rightarrow -\frac{\pi}{6} < 2x < \frac{5\pi}{6} \Rightarrow -\frac{1}{2} < \sin 2x < 1 \xrightarrow{\sin 2x = \frac{m-1}{4}} -\frac{1}{2} < \frac{m-1}{4} < 1$

$-2 < m-1 < 4 \xRightarrow{+1} -1 < m < 5$



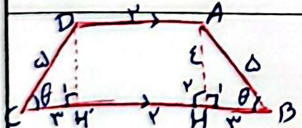
۳ (۲)

$\tan x + \cot x = \frac{1}{\sin x\cos x} = -3 \Rightarrow \sin x\cos x = -\frac{1}{3}$ $\quad \sin^2 x + \cos^2 x = 1$

$\Rightarrow \sin^2 x + \cos^2 x + 2\sin x\cos x = \frac{1}{3} \Rightarrow (\sin x + \cos x)^2 = \frac{1}{3} \Rightarrow \sin x + \cos x = \sqrt{\frac{1}{3}} \Rightarrow \sin^3 x + \cos^3 x =$

$(\sin x+\cos x)^3 - 3\sin x\cos x(\sin x + \cos x) = -\frac{1}{3}\sqrt{\frac{1}{3}} - 3(-\frac{1}{3})(-\sqrt{\frac{1}{3}}) = -\frac{4}{3}\sqrt{\frac{1}{3}} \Rightarrow \frac{1}{\sin^3 x + \cos^3 x} = -\frac{3}{4}\sqrt{3}$

۴ (۲)



$\hat{AHB}$: $\cos\theta = \frac{BH}{AB} = \frac{BH}{5} = \frac{6}{10} \Rightarrow BH = 3$ $\quad AH^2 + BH^2 = AB^2 \Rightarrow AH^2 = 16 \Rightarrow AH = 4$

$\left.\begin{array}{l} AB = DC \\ \hat{H}_1 = \hat{H'}_1 = 90^\circ \\ \hat{C} = \hat{B} = \theta \end{array}\right\} \xRightarrow{\text{وز}} A\hat{B}H \cong D\hat{C}H' \Rightarrow CH' = BH = 3$

$\left.\begin{array}{l} AD \parallel HH' \\ AH \parallel DH' \\ \hat{H}_2 = 90^\circ \end{array}\right\} \Rightarrow ADHH'$ مستطیل است $\Rightarrow AD = HH' = 2$

$S = \frac{AD + \overset{CH'+H'H+BH}{BC}}{2} \times AH = \frac{2+8}{2} \times 4 = 20$

۵ (۲)

$\tan(285^\circ)\tan(-165^\circ) - \sin(1095^\circ)\cos(255^\circ) = -\cot(15^\circ)\tan(15^\circ) - \sin(15^\circ)(-\sin 15^\circ)$

$\tan(285^\circ) = \tan(270^\circ + 15^\circ) = \tan(\frac{3\pi}{2} + 15) = -\cot 15^\circ$ $\quad = -1 + \sin^2(15^\circ) = -\cos^2 15^\circ$

$\tan(-165^\circ) = \tan(15^\circ - 180^\circ) = \tan(15^\circ - \pi) = \tan 15^\circ$

$\sin(1095^\circ) = \sin(1080^\circ + 15^\circ) = \sin(6\pi + 15^\circ) = \sin 15^\circ$ $\quad -\cos^2 15 = K\cos^2 15$

$\cos(255^\circ) = \cos(270^\circ - 15^\circ) = \cos(\frac{3\pi}{2} - 15^\circ) = -\sin 15^\circ$ $\quad \Rightarrow K = -1$

$A=\sqrt{۳}\cos(۲۱۰^\circ)\underbrace{\sin(۲۴۳^\circ)}_{-\cos(۲۷^\circ)}-\sqrt{۲}\sin(۱۳۵^\circ)\underbrace{\cos(۱۵۳^\circ)}_{-\cos ۲۷^\circ}$ $=\sqrt{۳}\left(\frac{-\sqrt{۳}}{۲}\right)(-\cos ۲۷^\circ)-\sqrt{۲}\left(\frac{\sqrt{۲}}{۲}\right)(-\cos ۲۷^\circ)=\frac{۳}{۲}\cos(۲۷^\circ)+\cos(۲۷^\circ)=\frac{۵}{۲}\cos(۲۷^\circ)$ $\frac{\frac{۵}{۲}\cos(۲۷^\circ)}{\cos(۲۷^\circ)}=\frac{۵}{۲}$ ✓	۶	۲
$f(x)=۱۶\cos^۲(۲x)\cos^۲(۴x)\cos^۲(۱۲x)\cos^۲(۲۴x)\xrightarrow{x=\frac{\pi}{۲۴}}$ ~~$f(\frac{\pi}{۲۴})$~~ $f\left(\frac{\pi}{۲۴}\right)=۱۶\cos^۲\left(\frac{\pi}{۱۲}\right)\cos^۲\left(\frac{\pi}{۶}\right)\cos^۲\left(\frac{\pi}{۳}\right)\cos^۲\left(\frac{۲\pi}{۳}\right)=۱۶\cos^۲(۱۵^\circ)\underbrace{\cos^۲(۳۰^\circ)}_{\frac{۳}{۴}}\underbrace{\cos^۲(۶۰^\circ)}_{\frac{۱}{۴}}\underbrace{\cos^۲(۱۲۰^\circ)}_{\frac{۱}{۴}}$ $۱۶\times\frac{۳}{۴}\times\frac{۱}{۴}\times\frac{۱}{۴}\times\cos^۲(۱۵^\circ)=\frac{۳}{۴}\cos^۲(۱۵^\circ)=\frac{۳}{۴}\times\frac{۲+\sqrt{۳}}{۴}=\frac{۶+۳\sqrt{۳}}{۱۶}$ $\cos^۲(۱۵^\circ)=۱-\sin^۲(۱۵^\circ)=۱-\frac{۲-\sqrt{۳}}{۴}=\frac{۲+\sqrt{۳}}{۴}$ $\hookrightarrow \sin^۲(۱۵^\circ)=\frac{۱-\cos ۳۰^\circ}{۲}=\frac{۱-\frac{\sqrt{۳}}{۲}}{۲}=\frac{۲-\sqrt{۳}}{۴}$	۷	۱/۵
$۱-\sin x=۴+۴\sin x\Rightarrow ۵\sin x=-۳\Rightarrow \sin x=\frac{-۳}{۵}\Rightarrow \sin^۲x=\frac{۹}{۲۵}\Rightarrow ۱-\sin^۲x=\frac{۱۶}{۲۵}$ $\Rightarrow\cos^۲x=\frac{۱۶}{۲۵}\Rightarrow\cos x=\pm\frac{۴}{۵}$ $\tan\frac{x}{۲}=\frac{\sin x}{۱+\cos x}=\frac{-\frac{۳}{۵}}{۱-\frac{۴}{۵}}=\frac{-\frac{۳}{۵}}{\frac{۱}{۵}}=-۳$ ✓ $\xRightarrow{x \text{ در ربع سوم}}\cos x=-\frac{۴}{۵}$	۸	۲
$\frac{\sin\theta}{۱-\cos\theta}=\frac{\cancel{۲}\cos\frac{\theta}{۲}\cancel{\sin\frac{\theta}{۲}}}{\cancel{۲}\sin^{\cancel{۲}}\frac{\theta}{۲}}=\frac{\cos\frac{\theta}{۲}}{\sin\frac{\theta}{۲}}=\cot\frac{\theta}{۲}$ $\frac{\sin\theta}{۱-\cos\theta}=\frac{\sin\theta(۱+\cos\theta)}{۱-\cos^۲\theta}=\frac{\cancel{\sin\theta}(۱+\cos\theta)}{\sin^{\cancel{۲}}\theta}=\frac{۱+\cos\theta}{\sin\theta}=\cot\frac{\theta}{۲}$ $\Big\}\Rightarrow$ $\frac{\sin\theta}{۱-\cos\theta}+\frac{۱+\cos\theta}{\sin\theta}=۲\cot\frac{\theta}{۲}=K\cot\frac{\theta}{۲}\Rightarrow K=۲$ ✓	۹	۲
$\sin\alpha=\frac{\sqrt{۲}}{۱۰}\Rightarrow\sin^۲\alpha=\frac{۲}{۱۰۰}\Rightarrow\cos^۲\alpha=\frac{۹۸}{۱۰۰}\Rightarrow\cos\alpha=\pm\frac{\sqrt{۹۸}}{۱۰}=\pm\frac{۷\sqrt{۲}}{۱۰}\xRightarrow{\alpha \text{ در ربع دوم}}\cos\alpha=-\frac{۷\sqrt{۲}}{۱۰}$ $\cos\left(\frac{۱۱\pi}{۴}+\alpha\right)=\underbrace{\cos\left(\frac{۱۱\pi}{۴}\right)}_{\cos\frac{۳\pi}{۴}}\cos(\alpha)-\underbrace{\sin\left(\frac{۱۱\pi}{۴}\right)}_{\sin(\frac{۳\pi}{۴})}\sin(\alpha)=\left(-\frac{\sqrt{۲}}{۲}\right)\left(-\frac{۷\sqrt{۲}}{۱۰}\right)-\left(\frac{\sqrt{۲}}{۲}\right)\left(\frac{\sqrt{۲}}{۱۰}\right)$ $=\frac{۱۴}{۲۰}-\frac{۲}{۲۰}=\frac{۱۲}{۲۰}=\frac{۳}{۵}$ ✓	۱۰	۲

-۷

$$f\left(\frac{\pi}{۳۶}\right) = ۱۶\cos^۲\left(\frac{\pi}{۱۲}\right)\cos^۲\left(\frac{\pi}{۶}\right)\cos^۲\left(\frac{\pi}{۳}\right)\cos^۲\left(\frac{۲\pi}{۳}\right)$$

$$\cos^۲\frac{\pi}{۱۲} = \frac{۱+\cancel{\cos\frac{\pi}{۶}}^{\frac{\sqrt{۳}}{۲}}}{۲} \longrightarrow \cos^۲\frac{\pi}{۱۲} = \frac{۲+\sqrt{۳}}{۴}$$

$$f\left(\frac{\pi}{۳۶}\right) = ۱۶\left(\frac{۲+\sqrt{۳}}{۴}\right) \times \frac{۳}{۴} \times \frac{۱}{۴} \times \frac{۱}{۴} = \boxed{\frac{۳(۲+\sqrt{۳})}{۱۶}}$$