تکلیف شماره ۲۸ — محسن نصری — ۱۵ — با زدهم بیرجند B

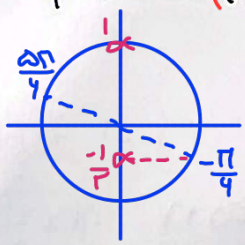
① $\cot\alpha = \frac{\cos\alpha}{|\sin\alpha|}$ ⇝ $\frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{|\sin\alpha|}$ ⇝ $\sin\alpha > 0$ ⇒ ناحیه اول یا ثانی

② $\frac{1}{|\cos\alpha|} - \frac{|\sin\alpha|}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|}$ (× $\frac{\cos\alpha}{\cos\alpha}$, $\sin\alpha \neq 1$)

$\frac{\cos\alpha - |\cos\alpha|\sin\alpha}{|\cos\alpha|\cos\alpha} = \frac{\cos\alpha - \cos\alpha\sin\alpha}{|\cos\alpha|\cos\alpha}$ ⇝ $\cos\alpha > 0$

③ $\frac{\pi}{12} < x < \frac{5\pi}{12}$ ⇝ $-\frac{\pi}{6} < 2x < \frac{5\pi}{6}$ ⇝ $-\frac{1}{2} < \sin 2x < 1$ ⇝ $-\frac{1}{2} < \frac{m-1}{4} < 1$

⇒ $-2 < m-1 < 4$ ⇝ $-1 < m < 5$ ✓ دقت!



④ $\tan\alpha + \cot\alpha < 0$ ($\tan\alpha\cdot\cot\alpha > 0$) ⇝ ناحیه دوم یا چهارم α (I)

$\pi < 4x < \frac{9\pi}{2}$ ⇝ ناحیه اول یا دوم x (II)

$\frac{\pi}{4} < x < \frac{\pi}{2}$

(I) ∩ (II) ⇒ x در ناحیه دوم

$\frac{\sin^2 x + \cos^2 x}{\sin x\cos x} = -2$ ⇝ $\sin x\cos x = -\frac{1}{2}$

$(\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2\sin x\cos x$

$(\sin x + \cos x)^2 = 1 - 1 = 0$ ⇒ $\sin x + \cos x = 0$

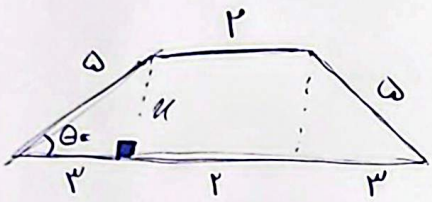
⑤ $(\sin x + \cos x)^3 = \sin^3 x + \cos^3 x + 3\sin x\cos x(\sin x + \cos x)$

$\sin^3 x + \cos^3 x = 0 + \frac{3\sqrt{2}}{2}$

$\frac{1}{\sin^3 x + \cos^3 x} = \frac{2\sqrt{2}}{3}$

$\frac{3\pi}{2} < 4x < 2\pi$ ⇝ $\frac{3}{4}\pi < x < \frac{\pi}{2}$ ($\sin x + \cos x < 0$) ⇝ $-\frac{\sqrt{2}}{2}$

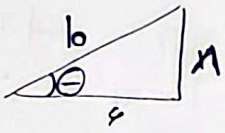
تکلیف شمارهٔ ۲۸ — معین نصری — یازدهم B



$\frac{x}{5} = \frac{8}{10} \rightarrow x = 4$

$\cos\theta = \frac{6}{10}$

$\sin\theta = \frac{8}{10}$



$S_{\text{ذوزنقه}} = \frac{4 \times (10)}{2} = 20$ ✓

(۴) ۲

$\tan(215^\circ)\tan(-145^\circ) - \sin(1080^\circ)\cos(200^\circ) = k\cos^2 15^\circ$

(۵)

$\rightarrow -\cot 15^\circ \times \tan 15^\circ - \sin 15^\circ \times (-\sin 15^\circ) = k\cos^2 15^\circ$

$-1 \quad + \sin^2 15^\circ$

$-\cos^2 15^\circ$

$\Rightarrow k = -1$

دقت!

۱,۷۵

$A = \sqrt{3}\cos(150^\circ)\sin(243^\circ) - \sqrt{2}\sin(135^\circ)\cos(153^\circ)$

(۶)

$\rightarrow \sqrt{3}\left(-\frac{\sqrt{3}}{2}\right) \times (-\cos 27^\circ) - \sqrt{2} \times \frac{\sqrt{2}}{2} \times (-\cos 27^\circ) = \frac{3}{2}\cos 27^\circ + \cos 27^\circ$

$= \frac{5}{2}\cos 27^\circ \xrightarrow{\div \cos 27^\circ} \frac{5}{2}$ ✓

۲

$f(x) = 14\cos^2(2x)\cos^2(3x)\cos^2(12x)\cos^2(24x)$

(۷)

$f\left(\frac{\pi}{24}\right) = ? \rightarrow 14 \times \cos^2\frac{\pi}{12} \times \cos^2\frac{\pi}{6} \times \cos^2\frac{\pi}{3} \times \cos^2\frac{12\pi}{3}$

$= 14 \times \frac{1+\frac{\sqrt{3}}{2}}{2} \times \left(\frac{\sqrt{3}}{2}\right)^2 \times \frac{1}{4} \times \frac{1}{4} = \frac{2+\sqrt{3}}{4} \times \sqrt{3} = \frac{2\sqrt{3}+3}{4}$

۱,۵

$1 - \sin x = 4 + 4\sin x \rightarrow 5\sin x = -3 \rightarrow \sin x = -\frac{3}{5}$ ، $\cos x = -\frac{4}{5}$ ربع سوم!

$\tan^2\frac{x}{2} = \frac{1-\cos x}{1+\cos x} \rightarrow \tan^2\frac{x}{2} = \frac{\frac{1}{5}}{\frac{9}{5}} = \frac{1}{9} \rightarrow \tan\frac{x}{2} = -\frac{1}{3}$

(۸)

۰,۷۵

$\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{-3}{5} \rightarrow 10\tan\frac{x}{2} = -3 - 3\tan^2\frac{x}{2}$

$\rightarrow \tan\frac{x}{2} = \frac{-1}{3}$ ✗ غیرصحیح!

$\rightarrow \tan\frac{x}{2} = -3$ ✓

تألیف شماره ۲۸ — محسن نصری — یازدهم سری B

$$\frac{\sin\theta}{1-\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=2\cot\frac{\theta}{2}\Rightarrow(K=2)$$

راه حل ۲!

۹) ۰٫۲۵

$$\cos\left(\frac{11\pi}{4}+\alpha\right)=\cos\left(2\pi+\frac{3\pi}{4}+\alpha\right)=\sin\left(\frac{3\pi}{4}+\alpha\right)$$

$$\Rightarrow \sin\left(\frac{\pi}{4}+\alpha\right)=\frac{\sin\alpha+\cos\alpha}{\sqrt{2}}=\frac{\frac{\sqrt{2}}{10}+\frac{\sqrt{98}}{10}}{\sqrt{2}}=\frac{1+7}{10}=\frac{8}{10}=\frac{4}{5}$$

۱۰) ۰٫۲۵

$$\frac{10}{\sqrt{98}}\sqrt{2}$$

-۷

$$f\left(\frac{\pi}{24}\right)=14\cos^2\left(\frac{\pi}{12}\right)\cos^2\left(\frac{\pi}{6}\right)\cos^2\left(\frac{\pi}{3}\right)\cos^2\left(\frac{2\pi}{3}\right)$$

$$\cos^2\frac{\pi}{12}=\frac{1+\cos\frac{\pi}{6}}{2}\left(\frac{\sqrt{3}}{2}\right)\rightarrow\cos^2\frac{\pi}{12}=\frac{2+\sqrt{3}}{4}$$

$$f\left(\frac{\pi}{24}\right)=14\left(\frac{2+\sqrt{3}}{4}\right)\times\frac{3}{4}\times\frac{1}{4}\times\frac{1}{4}=\boxed{\frac{3(2+\sqrt{3})}{14}}$$

-۹

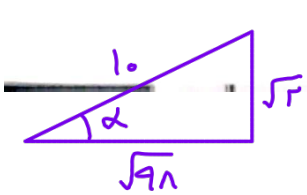
$$\frac{\sin\theta}{1-\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=\frac{\sin^2\theta+(1-\cos^2\theta)}{(1-\cos\theta)(\sin\theta)}=\frac{2\sin^2\theta}{\sin\theta(1-\cos\theta)}$$

$$\frac{2\sin\theta}{1-\cos\theta}=\frac{2\times 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}=2\cot\frac{\theta}{2}\rightarrow\boxed{K=2}$$

-۱۰

$$\cos\left(\frac{11\pi}{4}+\alpha\right)=\cos\left(2\pi-\frac{\pi}{4}+\alpha\right)=-\cos\left(\alpha-\frac{\pi}{4}\right)$$

$$=-\left(\cos\alpha\cos\frac{\pi}{4}+\sin\alpha\sin\frac{\pi}{4}\right)=-\frac{\sqrt{2}}{2}\left(\cos\alpha+\sin\alpha\right)$$



ناحیه دوم $\rightarrow\cos\alpha=\frac{-\sqrt{98}}{10}$

$$-\frac{\sqrt{2}}{2}\left(\cos\alpha+\sin\alpha\right)=-\frac{\sqrt{2}}{2}\left(-\frac{\sqrt{98}}{10}+\frac{\sqrt{2}}{10}\right)=\frac{3}{5}$$