نام و نام خانوادگی ...امیرمحمد مجتبائی....... پاسخنامه تشریحی تکلیف شماره ...۲۸... کلاس ...یازدهم ب.......

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در ناحیه اول است — ۲

$$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \Rightarrow \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{\cos x}$$

باید $\cos\alpha$ مثبت باشد تا تساوی برقرار باشد

$\cos\alpha > 0$ $\quad \frac{1-\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{\cos\alpha}$ ✓

$\cot\alpha = \frac{\cos\alpha}{|\sin\alpha|}$ —(باید $\sin\alpha$ مثبت باشد تا تساوی برقرار شود)→ $\sin\alpha > 0 \to \cot\alpha = \frac{\cos\alpha}{\sin\alpha}$ $\Big\} \Rightarrow \cos\alpha>0,\ \sin\alpha>0$

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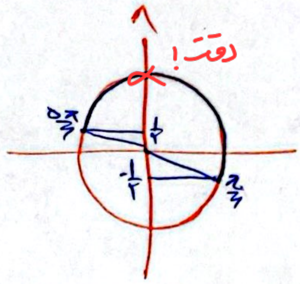
$$-\frac{\pi}{12} < x < \frac{5\pi}{12} \longrightarrow -\frac{\pi}{6} < 2x < \frac{5\pi}{6}$$

$$-\frac{1}{2} < \sin 2x \le 1 \xleftarrow{\sin 2x = \frac{m-1}{4}} -\frac{1}{2} < \frac{m-1}{4} \le 1$$

$$\Rightarrow -2 < m-1 \le 4 \Rightarrow -1 < m \le 5$$



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$\tan x + \cot x = -3$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x} = -3$$

$\Rightarrow \sin x \cos x = -\frac{1}{3}$

$(\sin x + \cos x)^2 = 1 + 2\sin x\cos x = 1 - \frac{2}{3} = \frac{1}{3} = (\sin x + \cos x)^2$

$|\sin x + \cos x| = \frac{1}{\sqrt{3}}$

ناحیه دوم $\frac{3}{4}\pi < x < \pi$, $\sin x + \cos x = -\frac{1}{\sqrt{3}}$

$$\frac{1}{\sin^3 x + \cos^3 x} = \frac{1}{(\sin x + \cos x)(\sin^2 x + \cos^2 x - \sin x\cos x)} = \frac{1}{-\frac{1}{\sqrt{3}}\left(1+\frac{1}{3}\right)} = \frac{-3\sqrt{3}}{4}$$

($\sin x+\cos x = -\frac{1}{\sqrt3}$, $\sin^2x+\cos^2x = 1$, $\sin x\cos x = -\frac13$, $1+\frac13 = \frac43$)

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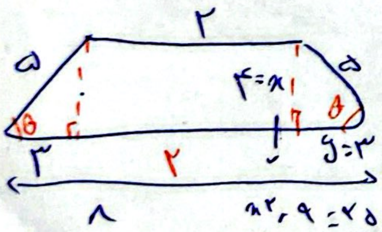
$\cos\theta = 0.6$



$\frac{y}{5} = 0.6 \Rightarrow y = 3$

$x^2 + 9 = 25$

$x = 4$

$$S = \frac{(2+8)\times 4}{2} = 20$$ ✓

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$$\tan(285^\circ)\tan(-165^\circ) - \sin(1905)\cos(255^\circ) = \tan\left(\frac{3\pi}{2}+15\right)\tan(-\pi+15) - \sin(6\pi+105)\left(\cos\left(\frac{3\pi}{2}-15\right)\right) = -\cot(15)(\tan(15)) + \sin(75^\circ)\sin(15^\circ)$$

$$-1 + \sin^2 15 = -(1-\sin^2 15) = -(\cos^2 15) = k\cos^2 15$$

$$k = -1$$ ✓

$\sqrt{3}\cos(210)\sin(243) - \sqrt{2}\sin(135^\circ)\cos(153) = \sqrt{3}\cos(\frac{3\pi}{2} - 30)\sin(\frac{3\pi}{2} - 27) - \sqrt{2}\sin(135)\cos(\pi - 27) = +\frac{\sqrt{3}}{2}\cdot\sqrt{3}\cos 30 \times \cos 27 + \sqrt{2}\cdot\frac{\sqrt{2}}{2}\cos 27 =$ $\frac{3}{2}\cos 27 + \cos 27 = \frac{5}{2}\cos 27 = k\cos 27^\circ \Rightarrow \boxed{k = \frac{5}{2}}$ ✓	(۲)	۶
$f(x) = 16\cos^2(3x)\cos^2(6x)\cos^2(12x)\cos^2(24x)$ $f(\frac{\pi}{24}) = 16\cos^2(\frac{\pi}{12})\cos^2\frac{\pi}{6}\cos^2(\frac{\pi}{3})\cos^2\frac{2\pi}{3}$ $\cos^2\frac{\pi}{12} = \frac{1+\cos\frac{\pi}{6}}{2} = \frac{1+\frac{\sqrt{3}}{2}}{2} = \frac{\frac{2+\sqrt{3}}{2}}{2} = \frac{2+\sqrt{3}}{4}$ $16 \times \frac{2+\sqrt{3}}{4} \times (\frac{\sqrt{3}}{2})^2 \times (\frac{1}{2})^2 \times (-\frac{1}{2})^2 = 4 \times (2+\sqrt{3}) \times \frac{3}{4} \times \frac{1}{4} \times \frac{1}{4} = \boxed{\frac{6+3\sqrt{3}}{16}}$ ✓	(۲)	۷
$\frac{1-\sin x}{1+\sin x} = 4 \quad 4 + 4\sin x = 1 - \sin x \Rightarrow 5\sin x = -3 \Rightarrow \sin x = \frac{-3}{5}$ $\cos x = \frac{4}{5}$ ربع سوم $, \cos x = -\frac{4}{5} \qquad \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$ $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2} \qquad \tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\frac{3}{\sqrt{10}}}{-\frac{1}{\sqrt{10}}} = -3$ اگر x در ناحیه سوم باشد $\frac{x}{2}$ هم در ناحیه دوم است $\cos^2\frac{x}{2} = \frac{1+\cos x}{2} = \frac{1-\frac{4}{5}}{2} = \frac{\frac{1}{5}}{2} = \frac{1}{10} \Rightarrow \cos\frac{x}{2} = -\frac{1}{\sqrt{10}} \Rightarrow \sin\frac{x}{2} = \frac{3}{\sqrt{10}}$	(۱٫۵)	۸
$\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \sin^2\theta}{\sin\theta(1-\cos\theta)} = \frac{2\sin^2\theta}{\sin\theta(1-\cos\theta)} = \frac{2\sin\theta}{1-\cos\theta} = \frac{2\sin\theta}{2\sin^2\frac{\theta}{2}} = \frac{4\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} = \frac{2\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}$ $2\cot\theta = k\cot\theta \Rightarrow \boxed{k=2}$ ✓	(۲)	۹
$\sin\alpha = \frac{\sqrt{2}}{10}$ ناحیه دوم $, \cos\alpha = -\frac{\sqrt{98}}{10} = \frac{-7\sqrt{2}}{10}$ $\cos(\frac{11\pi}{4} + \alpha) = \cos(2\pi + \frac{3\pi}{4} + \alpha) = \cos(\frac{3\pi}{4} + \alpha) = \cos\frac{3\pi}{4}\cos\alpha - \sin\frac{3\pi}{4}\sin\alpha =$ $-\frac{\sqrt{2}}{2} \times -\frac{7\sqrt{2}}{10} - \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{10} = \frac{14}{20} - \frac{2}{20} = \frac{7}{10} - \frac{1}{10} = \frac{6}{10} = \boxed{0.6}$ ✓	(۲)	۱۰

$$\sin\alpha = \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \frac{-3}{5} \rightarrow 10\tan\frac{\alpha}{2} = -6 - 3\tan^2\frac{\alpha}{2}$$

$\tan\frac{\alpha}{2} = \frac{-1}{3}$ غ‌ق‌ق! ×

$\boxed{\tan\frac{\alpha}{2} = -3}$ ✓