

به نام خدا

بیر B

$$\cot \alpha = \frac{\cos \alpha}{\sqrt{1 - \sin^2 \alpha}} = \frac{\cos \alpha}{\sqrt{\sin^2 \alpha}} = \frac{\cos \alpha}{|\sin \alpha|}$$

۲۰ آفرین!

$$\Rightarrow \cot \alpha = \frac{\cos \alpha}{\sin \alpha} = \frac{\cos \alpha}{|\sin \alpha|} \Rightarrow \sin \alpha = |\sin \alpha| \Rightarrow \sin \alpha > 0$$

$$\frac{1}{\sqrt{\cos^2 \alpha}} - \frac{1}{\cot \alpha} = \frac{1}{|\cos \alpha|} - \frac{\sin \alpha}{\cos \alpha} = \frac{1 - \sin \alpha}{|\cos \alpha|}$$

(۱) (۲)

$$\Rightarrow \frac{1}{|\cos \alpha|} - \frac{\sin \alpha}{\cos \alpha} = \frac{1}{|\cos \alpha|} - \frac{\sin \alpha}{|\cos \alpha|} \Rightarrow \frac{\sin \alpha}{\cos \alpha} = \frac{\sin \alpha}{|\cos \alpha|}$$

$$\Rightarrow \cos \alpha = |\cos \alpha| \Rightarrow \cos \alpha > 0$$

$\begin{cases} \sin \alpha > 0 \\ \cos \alpha > 0 \end{cases} \Rightarrow$  در ناحیه اول قرار دارد ✓

$$-\frac{\pi}{2} < \alpha < \frac{\pi}{2} \Rightarrow -\frac{\pi}{4} < \alpha < \frac{\pi}{4}$$

$$-\frac{1}{\mu} < \sin t \leq 1 \leftarrow -\frac{\pi}{2} < t < \frac{\pi}{2} \text{ است } t = \alpha \text{ پس } -\frac{1}{\mu} < \sin \alpha \leq 1$$

$$\Rightarrow \sin \alpha = \frac{m-1}{\mu} \Rightarrow -\frac{1}{\mu} < \frac{m-1}{\mu} \leq 1 \Rightarrow -\mu < m-1 \leq \mu$$

$$\Rightarrow -1 < m \leq \mu \Rightarrow m \in (-1, \mu] \checkmark$$

(۲) (۳)

$$\tan \alpha + \cot \alpha = -\mu \Rightarrow \frac{\sin \alpha}{\cos \alpha} + \frac{\cos \alpha}{\sin \alpha} = \frac{1}{\sin \alpha \cdot \cos \alpha} = -\mu$$

$$\Rightarrow \sin \alpha \cdot \cos \alpha = -\frac{1}{\mu}$$

(۲) (۳)

$$A = \sin \alpha + \cos \alpha \Rightarrow A^2 = (\sin \alpha + \cos \alpha)^2 = \sin^2 \alpha + \cos^2 \alpha + 2 \sin \alpha \cos \alpha$$

$$\Rightarrow A^2 = 1 - \frac{2}{\mu} = \frac{\mu - 2}{\mu} \Rightarrow \begin{cases} A = \frac{1}{\sqrt{\mu}} \\ A = -\frac{1}{\sqrt{\mu}} \end{cases}$$

$$3\pi < \theta < 4\pi \Rightarrow \frac{3\pi}{4} < \alpha < \pi$$

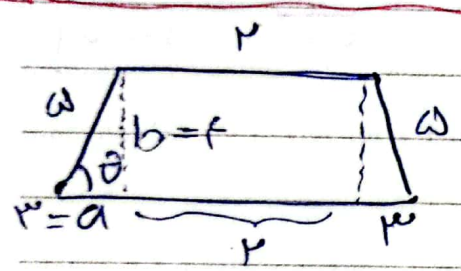
زاویه  $\alpha$  نه ناصبه دوام مثلثات قرار دارد که  $\cos \alpha < 0$  و  $\sin \alpha > 0$  (دوم و سوم)

$A = \sin \alpha + \cos \alpha$  در نتیجه مقدار  $|\sin \alpha| < |\cos \alpha|$  قرار دارد  $\frac{3\pi}{4} < \alpha < \pi$

پایه منفی باشد به نتیجه  $\sin \alpha + \cos \alpha = A = \frac{-1}{\sqrt{3}}$

$$\frac{1}{\cos^2 \alpha + \sin^2 \alpha} = \frac{1}{A(1 - (-\frac{1}{\sqrt{3}}))} = \frac{1}{-\frac{1}{\sqrt{3}} \times \frac{4}{3}} = \frac{1}{-\frac{4}{3\sqrt{3}}} = \frac{-3\sqrt{3}}{4}$$

(3)



$$\cos \theta = \frac{a}{5} = 0.4 \Rightarrow a = 2$$

$$a^2 + b^2 = 5^2 \Rightarrow 4 + b^2 = 25 \Rightarrow b = 3$$

ارتفاع = 4      قاعده بزرگ = 3      قاعده کوچک = 2

(2) (4)

مساحت ذوزنقه  $S = \frac{(2+3)}{2} \times 4 = 10$

من دانیم:  $\tan(-\theta) = -\tan \theta$

$$\tan(28^\circ) \tan(-14^\circ) - \sin(109^\circ) \Rightarrow (28^\circ)$$

$$= -\tan(28^\circ) \tan(14^\circ) - \sin(109^\circ) \Rightarrow (28^\circ)$$

$$= -\tan(27^\circ + 1^\circ) \tan(18^\circ - 1^\circ) - \sin(108^\circ + 1^\circ) \Rightarrow (27^\circ - 1^\circ)$$

$$= \cot 1^\circ (-\tan 1^\circ) - \sin 1^\circ (-\sin 1^\circ) = -1 + \sin^2 1^\circ = -\cos^2 1^\circ$$

$$\Rightarrow K \cos^2 1^\circ = -\cos^2 1^\circ \Rightarrow K = -1$$

(2) (3)

$$A = \sqrt{3} \cos(110^\circ) \sin(143^\circ) - \sqrt{2} \sin(135^\circ) \cos(152^\circ)$$

$$= \sqrt{3} \cos(180^\circ + 30^\circ) \sin(180^\circ - 37^\circ) - \sqrt{2} \sin(90^\circ + 45^\circ) \cos(180^\circ - 37^\circ)$$

$$= \sqrt{3} (-\cos 30^\circ) (-\cos 37^\circ) - \sqrt{2} (\sin 45^\circ) (-\cos 37^\circ)$$

$$= \sqrt{3} \left(\frac{\sqrt{3}}{2}\right) (\cos 37^\circ) + \sqrt{2} \left(\frac{\sqrt{2}}{2}\right) (\cos 37^\circ) = \frac{5}{2} (\cos 37^\circ)$$

حاصل عبارت A برابر  $\frac{5}{2} \cos 37^\circ$  است ✓

$$f\left(\frac{\pi}{12}\right) = 14 \cos^2 \frac{3\pi}{12} \cos^2 \frac{4\pi}{12} \cos^2 \frac{13\pi}{12} \cos^2 \frac{14\pi}{12}$$

$$= 14 \cos^2 \frac{\pi}{12} \cos^2 \frac{\pi}{6} \cos^2 \frac{\pi}{12} \cos^2 \frac{7\pi}{12} = 14 \cos^2 \frac{\pi}{12} \left(\frac{\sqrt{3}}{2}\right)^2 \left(\frac{1}{2}\right)^2 \left(-\frac{1}{2}\right)^2$$

$$= \frac{3}{2} \cos^2 \frac{\pi}{12}$$

اکنون برای این مقدار  $\cos^2 \frac{\pi}{12}$  را حساب کنیم از اتحاد مثلثات  $\cos^2 \frac{\alpha}{2} = \frac{1 + \cos \alpha}{2}$  استفاده می‌کنیم. بنابراین:

$$f\left(\frac{\pi}{12}\right) = \frac{3}{2} \times \frac{1}{2} \left(1 + \cos \frac{\pi}{6}\right) = \frac{3}{4} \left(1 + \frac{\sqrt{3}}{2}\right) = \frac{4 + 3\sqrt{3}}{8}$$

$$\frac{1 - \sin \alpha}{1 + \sin \alpha} = f \Rightarrow 1 - \sin \alpha = f(1 + \sin \alpha) \Rightarrow 2 \sin \alpha = -f \Rightarrow \sin \alpha = -\frac{f}{2}$$

$$\Rightarrow \sin \alpha = \frac{2 \tan \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} = -\frac{f}{2} \Rightarrow -f(1 + \tan^2 \frac{\alpha}{2}) = 2 \tan \frac{\alpha}{2}$$

$$\Rightarrow 2 \tan^2 \frac{\alpha}{2} + 2 \tan \frac{\alpha}{2} + f = 0 \Rightarrow \tan \frac{\alpha}{2} = \frac{-1 \pm \sqrt{1 - \frac{f^2}{4}}}{1}$$

$$\Rightarrow \tan \frac{\alpha}{2} = \frac{-1 \pm \sqrt{4 - f^2}}{2} = \frac{-1 \pm 1}{2} \Rightarrow \begin{cases} \tan \frac{\alpha}{2} = -\frac{1}{2} \\ \tan \frac{\alpha}{2} = -1 \end{cases}$$

مقدار صحیح  $\tan \frac{\alpha}{2} = -1$  ✓

مقدار صحیح  $\tan \frac{\alpha}{2}$  برابر  $-1$  است.

$$\sin \theta = r \sin \frac{\theta}{r} \cos \frac{\theta}{r} \quad , \quad r \cos \frac{\theta}{r} = 1 + \cos \theta \quad , \quad r \sin \frac{\theta}{r} = 1 - \cos \theta$$

$$\frac{\sin \theta}{1 - \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = \frac{r \sin \frac{\theta}{r} \cos \frac{\theta}{r}}{r \sin \frac{\theta}{r}} + \frac{r \cos \frac{\theta}{r}}{r \sin \frac{\theta}{r} \cos \frac{\theta}{r}} \quad (r)$$

$$= \cot \frac{\theta}{r} + \cot \frac{\theta}{r} = 2 \cot \frac{\theta}{r} = k \cot \frac{\theta}{r} \Rightarrow \underline{k = 2}$$

$$\sin \alpha = \frac{\sqrt{r}}{10} \rightarrow \sin^2 \alpha + \cos^2 \alpha = 1 \Rightarrow \frac{r}{100} + \cos^2 \alpha = 1 \Rightarrow \cos^2 \alpha = \frac{9r}{100}$$

$$\Rightarrow \cos^2 \alpha = \frac{9r}{100} \Rightarrow \cos \alpha = \frac{-\sqrt{r}}{10} \rightarrow \text{زیا در ربع دوم مثلثاتی قرار دارد}$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\Rightarrow \cos\left(\frac{11\pi}{r} + \alpha\right) = \cos \frac{11\pi}{r} \cos \alpha - \sin \alpha \sin \frac{11\pi}{r} \quad (r)$$

$$\cos \frac{11\pi}{r} = \cos\left(2\pi + \frac{3\pi}{r}\right) = \cos\left(\frac{3\pi}{r}\right) = \frac{-\sqrt{r}}{r} \Rightarrow \sin \frac{3\pi}{r} = \frac{\sqrt{r}}{r}$$

$$\Rightarrow \cos \frac{3\pi}{r} \cos \alpha - \sin \alpha \sin \frac{3\pi}{r} = \left(\frac{-\sqrt{r}}{r}\right) \left(\frac{-\sqrt{r}}{10}\right) - \left(\frac{\sqrt{r}}{10}\right) \left(\frac{\sqrt{r}}{r}\right)$$

$$= \frac{r}{10} - \frac{1}{10} = \frac{r-1}{10} \Rightarrow \cos\left(\frac{11\pi}{r} + \alpha\right) = \frac{r-1}{10}$$