به نام خدا

پیر B

Subject:

Year: Month: Day: ( ) ونداد حاجی باباش - تکلیف سری ۲۸ - یازدهم page: ( )

$$\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} = \frac{\cos\alpha}{\sqrt{\sin^2\alpha}} = \frac{\cos\alpha}{|\sin\alpha|}$$

$$\Rightarrow \cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow \sin\alpha = |\sin\alpha| \Rightarrow \sin\alpha > 0$$

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$$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|}$$

$$\Rightarrow \cancel{\frac{1}{|\cos\alpha|}} - \frac{\sin\alpha}{\cos\alpha} = \cancel{\frac{1}{|\cos\alpha|}} - \frac{\sin\alpha}{|\cos\alpha|} \Rightarrow \frac{\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{|\cos\alpha|}$$

$$\Rightarrow \cos\alpha = |\cos\alpha| \Rightarrow \cos\alpha > 0$$

$\begin{cases} \sin\alpha > 0 \\ \cos\alpha > 0 \end{cases} \Rightarrow$ α در ناحیه اول قرار دارد

$$-\frac{\pi}{12} < x < \frac{5\pi}{12} \Rightarrow -\frac{\pi}{6} < 2x < \frac{5\pi}{6}$$

پس اگر $t = 2x$ ، آنگاه $-\frac{\pi}{6} < t < \frac{5\pi}{6} \Rightarrow -\frac{1}{2} < \sin t \leq 1$

$$\Rightarrow \sin 2x = \frac{m-1}{4} \xrightarrow{-\frac{1}{2} < \sin 2x \leq 1} -\frac{1}{2} < \frac{m-1}{4} \leq 1 \Rightarrow -2 < m-1 \leq 4$$

$$\Rightarrow -1 < m \leq 5 \Rightarrow m \in (-1, 5]$$

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$$\tan x + \cot x = -3 \Rightarrow \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{1}{\sin x \cdot \cos x} = -3$$

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$$\Rightarrow \sin x \cdot \cos x = \frac{-1}{3}$$

$$A = \sin x + \cos x \Rightarrow A^2 = (\sin x + \cos x)^2 = \underbrace{\sin^2 x + \cos^2 x}_{1} + 2\underbrace{\sin x\cos x}_{-\frac{1}{3}}$$

$$\Rightarrow A^2 = 1 - \frac{2}{3} = \frac{1}{3} \Rightarrow \begin{cases} A = \frac{1}{\sqrt{3}} \\ A = \frac{-1}{\sqrt{3}} \end{cases}$$

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$3\pi < 4x < 4\pi \Rightarrow \frac{3}{4}\pi < x < \pi$

زاویه x در ناحیه دوم مثلثاتی قرار دارد که $\cos x < 0$ و $\sin x > 0$ ولی چون

$\frac{3\pi}{4} < x < \pi$ قرار دارد $|\sin x| < |\cos x|$ در نتیجه مقدار $A = \sin x + \cos x$

باید منفی باشد در نتیجه $\sin x + \cos x = A = \frac{-1}{\sqrt{3}}$

$$\frac{1}{\cos^3 x + \sin^3 x} = \frac{1}{A(1-(-\frac{1}{3}))} = \frac{1}{-\frac{1}{\sqrt{3}} \times \frac{4}{3}} = \frac{1}{\frac{-4}{3\sqrt{3}}} = \frac{-3\sqrt{3}}{4}$$

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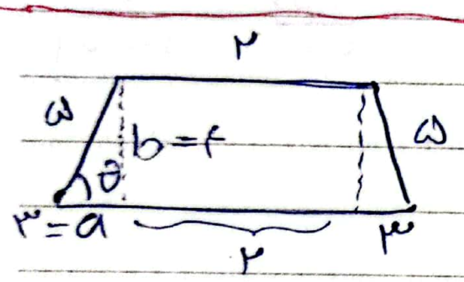

$\cos\theta = \frac{a}{5} = 0.6 \Rightarrow \boxed{a=3}$

$a^2+b^2=5^2 \Rightarrow 9+b^2=25 \Rightarrow \boxed{b=4}$

قاعده کوچک = ۲   قاعده بزرگ = ۸   ارتفاع = ۴

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$S_{\text{ذوزنقه}} = \frac{(\text{قاعده کوچک} + \text{قاعده بزرگ})}{2} \times \text{ارتفاع} = \frac{(2+8)}{2} \times 4 = \boxed{20} \rightarrow$ مساحت ذوزنقه

می‌دانیم: $\tan(-\theta) = -\tan\theta$

$\tan(285^\circ)\tan(-165^\circ) - \sin(1095^\circ)\cos(255^\circ)$

$= -\tan(285^\circ)\tan(165^\circ) - \sin(1095^\circ)\cos(255^\circ)$

$= -\tan(270^\circ+15^\circ)\tan(180^\circ-15^\circ) - \sin(1080^\circ+15^\circ)\cos(270^\circ-15^\circ)$

$= \cot 15^\circ(-\tan 15^\circ) - \sin 15^\circ(-\sin 15^\circ) = -1 + \sin^2 15^\circ = -\cos^2 15^\circ$

$\Rightarrow K\cos^2 15^\circ = -\cos^2 15^\circ \Rightarrow \boxed{K=-1}$

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$$A = \sqrt{3}\cos(210^\circ)\sin(243^\circ) - \sqrt{2}\sin(135^\circ)\cos(153^\circ)$$

$$= \sqrt{3}\cos(180^\circ+30^\circ)\sin(270^\circ-27^\circ) - \sqrt{2}\sin(90^\circ+45^\circ)\cos(180^\circ-27^\circ)$$

$$= \sqrt{3}(-\cos 30^\circ)(-\cos 27^\circ) - \sqrt{2}(\sin 45^\circ)(-\cos 27^\circ)$$

$$= \sqrt{3}\left(\frac{\sqrt{3}}{2}\right)(\cos 27^\circ) + \sqrt{2}\left(\frac{\sqrt{2}}{2}\right)(\cos 27^\circ) = \frac{5}{2}(\cos 27^\circ)$$

حاصل عبارت A ، $\frac{5}{2}$ برابر $\cos 27^\circ$ است.

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$$f\left(\frac{\pi}{36}\right) = 16\cos^2\frac{3\pi}{36}\cos^2\frac{6\pi}{36}\cos^2\frac{12\pi}{36}\cos^2\frac{24\pi}{36}$$

$$= 16\cos^2\frac{\pi}{12}\cos^2\frac{\pi}{6}\cos^2\frac{\pi}{3}\cos^2\frac{2\pi}{3} = 16\cos^2\frac{\pi}{12}\left(\frac{\sqrt{3}}{2}\right)^2\left(\frac{1}{2}\right)^2\left(-\frac{1}{2}\right)^2$$

$$= \frac{3}{4}\cos^2\frac{\pi}{12}$$

اکنون برای اینکه مقدار $\cos^2\frac{\pi}{12}$ را حساب کنیم از اتحاد مثلثاتی $\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$ استفاده می‌کنیم، بنابراین:

$$f\left(\frac{\pi}{36}\right) = \frac{3}{4}\times\frac{1}{2}\left(1+\cos\frac{2\pi}{12}\right) = \frac{3}{8}\left(1+\frac{\sqrt{3}}{2}\right) = \frac{6+3\sqrt{3}}{16}$$

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$$\frac{1-\sin x}{1+\sin x} = 4 \Rightarrow 1-\sin x = 4(\sin x+1) \Rightarrow 5\sin x = -3 \Rightarrow \sin x = \frac{-3}{5}$$

$$\Rightarrow \sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{-3}{5} \Rightarrow -3\left(1+\tan^2\frac{x}{2}\right) = 10\tan\frac{x}{2}$$

$$\Rightarrow 3\tan^2\frac{x}{2} + 10\tan\frac{x}{2} + 3 = 0 \Rightarrow \tan\frac{x}{2} = \frac{-10\pm\sqrt{100-36}}{6}$$

$$\Rightarrow \tan\frac{x}{2} = \frac{-10\pm\sqrt{64}}{6} = \frac{-10\pm 8}{6} \Rightarrow \begin{cases} \tan\frac{x}{2} = \frac{-1}{3} \\ \tan\frac{x}{2} = -3 \longrightarrow \text{مقدار صحیح} \end{cases}$$

مقدار صحیح $\tan\frac{x}{2}$ برابر -3 می‌باشد.

$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$ ، $2\cos^2\frac{\theta}{2} = 1+\cos\theta$ ، $2\sin^2\frac{\theta}{2} = 1-\cos\theta$

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$$\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} + \frac{2\cos^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$= \cot\frac{\theta}{2}+\cot\frac{\theta}{2} = 2\cot\frac{\theta}{2} = k\cot\frac{\theta}{2} \Rightarrow k=2$$

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$$\sin\alpha = \frac{\sqrt{2}}{10} \longrightarrow \sin^2\alpha+\cos^2\alpha=1 \Rightarrow \frac{2}{100}+\cos^2\alpha=1 \Rightarrow \cos^2\alpha=\frac{98}{100}$$

$\Rightarrow \cos^2\alpha = \frac{49}{50} \Rightarrow \cos\alpha = \frac{-7}{5\sqrt{2}} \longrightarrow$ زیرا α در ربع دوم مثلثاتی قرار دارد

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\Rightarrow \cos\left(\frac{11\pi}{4}+\alpha\right)=\cos\frac{11\pi}{4}\cdot\cos\alpha-\sin\alpha\sin\frac{11\pi}{4}$$

$$\cos\frac{11\pi}{4}=\cos\left(2\pi+\frac{3\pi}{4}\right)=\cos\left(\frac{3\pi}{4}\right)=\frac{-\sqrt{2}}{2} \Rightarrow \sin\frac{3\pi}{4}=\frac{\sqrt{2}}{2}$$

$$\Rightarrow \cos\frac{3\pi}{4}\cdot\cos\alpha-\sin\alpha\sin\frac{3\pi}{4}=\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{-7}{5\sqrt{2}}\right)-\left(\frac{\sqrt{2}}{10}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$=\frac{7}{10}-\frac{1}{10}=\frac{6}{10} \Rightarrow \cos\left(\frac{11\pi}{4}+\alpha\right)=\frac{6}{10}$$