محمد شیرازی ۹۱/B

۱- (۲)

$\frac{\cos a}{\sin a} = \frac{\cos a}{|\sin a|} \Rightarrow \sin a > 0$; $0 < a < \frac{\pi}{2} \to$ ناحیه اول

$\frac{1}{|\cos a|} - \frac{\sin a}{\cos a} = \frac{1-\sin a}{\cos a} \to \cos a > 0$

۲- (۲) ۱۸٫۵

$-\frac{\pi}{6} < 2x < \frac{7\pi}{6}$

$-\frac{1}{2} < \sin 2x \leq 1$

$-\frac{1}{2} < \frac{m-1}{4} \leq 1$

$-1 < m \leq 5$

۳- (۱٫۵)

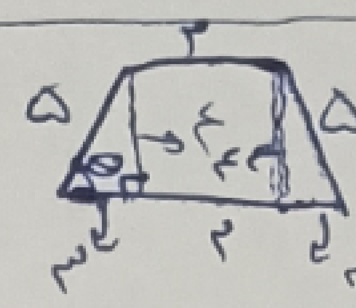
$\tan x + \cot x = \frac{2}{\sin 2x} = -3 \to \sin 2x = -\frac{2}{3}$

$(\sin x + \cos x)^2 = 1 + \sin 2x = 1 - \frac{2}{3} = \frac{1}{3} \to \sin x + \cos x = \frac{\sqrt{3}}{3}$

$\sin^3 x + \cos^3 x = (\sin + \cos)(\cos^2 + \sin^2 - \sin\cos) = \frac{\sqrt{3}}{3} \times \frac{4}{3} = \frac{4\sqrt{3}}{9}$

$\frac{1}{\sin^3 x + \cos^3 x} = \frac{1}{\frac{4\sqrt{3}}{9}} = \frac{9}{4\sqrt{3}} = \frac{3\sqrt{3}}{4}$

۴- (۲)



$3 + 3 + 2 = 8$

$\frac{(8+2)4}{2} = 20$

۵- (۲)

$\tan 285° = \tan(270 + 15) = -\cot 15°$

$\tan(-165°) = \tan(-180 + 15) = -\tan(180 - 15) = \tan 15$

$\sin(1095°) = \sin(3 \times 360 + 15) = \sin 15$

$\cos 255° = \cos(270° - 15°) = -\sin 15°$

$-\cot 15 \times \tan 15 - \sin 15 \times -\sin 15 = -1 + \sin^2 15 = -\cos^2 15°$

$\boxed{K = -1}$

۶- (۲)

$\cos 210 = -\frac{\sqrt{3}}{2}$; $\sin(243°) = \sin(\frac{3\pi}{2} - 27) = -\cos 27$

$\sin 135 = \frac{\sqrt{2}}{2}$; $\cos(153°) = \cos(\pi - 27) = -\cos 27$

$\Rightarrow \sqrt{3} \times \frac{-\sqrt{3}}{2} \times -\cos 27° - \sqrt{2} \times \frac{\sqrt{2}}{2} \times -\cos 27$

$= -\cos 27°(-\frac{3}{2} - 1) \to \frac{-\cos 27°(-\frac{3}{2} - 1)}{\cos 27°} = \frac{3}{2} + 1 = \frac{5}{2} = 2.5$

۷- (۱٫۵)

$8(2\cos^2 15) = 8(\cos 30 + 1) = 8(\frac{\sqrt{3} + 2}{4}) = (2\sqrt{3} + 4)$

$(2\sqrt{3} + 4) \times (\frac{\sqrt{3}}{2})^2 \times (\frac{1}{2})^2 \times (-\frac{1}{2})^2 = (2\sqrt{3} + 4) \times \frac{3}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{3(2\sqrt{3} + 4)}{64}$

۸- (۲)

$1 - \sin x = 4 + 4\sin x \to \sin x = -\frac{3}{5} \to \cos x = -\frac{4}{5} \to \tan x = \frac{3}{4}$

$\tan x = \frac{2\tan\frac{x}{2}}{1 - \tan^2\frac{x}{2}} \Rightarrow \tan\frac{x}{2} = -3$

۹- (۲)

$\frac{\sin^2\theta + 1 - \cos^2\theta}{\sin\theta(1 - \cos\theta)} = \frac{2\sin^2\theta}{\sin\theta(1 - \cos\theta)} = \frac{2\sin\theta}{1 - \cos\theta}$

$= \frac{4\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} = \frac{2\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} = 2\cot\frac{\theta}{2}$

$K = 2$

۱۰- (۱٫۵)

$\cos(\frac{11\pi}{4} + a) = \cos(\frac{3\pi}{4} + a)$

$\cos\frac{3\pi}{4} \cdot \cos a - \sin\frac{3\pi}{4} \sin a =$

$\boxed{\cos = -\sqrt{1 - \frac{2}{100}} = \frac{-\sqrt{98}}{10}}$

$\frac{-\sqrt{2}}{2} \cdot \frac{-\sqrt{98}}{10} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{10} = \frac{-\sqrt{2}}{2}(-\frac{\sqrt{98}}{10} - \frac{\sqrt{2}}{10}) = \frac{\sqrt{196}}{20} + \frac{2}{20} = \frac{14 + 2}{20} = \frac{16}{20} = \frac{4}{5}$

$= 0.8$

-۳

$$(\sin\alpha+\cos\alpha)^{۲} = ۱ + ۲\sin\alpha\cos\alpha$$

$$= ۱ + ۲\left(\frac{-۱}{۳}\right) = \frac{۱}{۳}$$

$$۳\pi < ۴x < ۴\pi \to \frac{۳}{۴}\pi < x < \pi \xrightarrow{\sin x+\cos x<۰} \frac{-\sqrt{۳}}{۳}$$

$$\sin^{۳}x+\cos^{۳}x = (\sin x+\cos x)(\sin^{۲}x+\cos^{۲}x-\sin x\cos x) = -\frac{\sqrt{۳}}{۳}\left(\frac{۴}{۳}\right)$$

$$۱-\left(-\frac{۱}{۳}\right) = \frac{۴}{۳}$$

$$\to \frac{۱}{\sin^{۳}x+\cos^{۳}x} = \boxed{\frac{-۳}{۴}\sqrt{۳}}$$

-۷

$$f\left(\frac{\pi}{۲۴}\right) = ۱۶\cos^{۲}\left(\frac{\pi}{۱۲}\right)\cos^{۲}\left(\frac{\pi}{۶}\right)\cos^{۲}\left(\frac{\pi}{۳}\right)\cos^{۲}\left(\frac{۲\pi}{۳}\right)$$

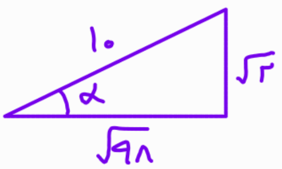
$$\cos^{۲}\frac{\pi}{۱۲} = \frac{۱+\cancelto{\frac{\sqrt{۳}}{۲}}{\cos\frac{\pi}{۶}}}{۲} \to \cos^{۲}\frac{\pi}{۱۲} = \frac{۲+\sqrt{۳}}{۴}$$

$$f\left(\frac{\pi}{۲۴}\right) = ۱۶\left(\frac{۲+\sqrt{۳}}{۴}\right) \times \frac{۳}{۴} \times \frac{۱}{۴} \times \frac{۱}{۴} = \boxed{\frac{۳(۲+\sqrt{۳})}{۱۶}}$$

-۱۰

$$\cos\left(\frac{۱۱\pi}{۴}+\alpha\right) = \cos\left(۳\pi-\frac{\pi}{۴}+\alpha\right) = -\cos\left(\alpha-\frac{\pi}{۴}\right)$$

$$= -\left(\cos\alpha\cos\frac{\pi}{۴}+\sin\alpha\sin\frac{\pi}{۴}\right) = -\frac{\sqrt{۲}}{۲}(\cos\alpha+\sin\alpha)$$



$$\xrightarrow{\text{ناحیه دوم}} \cos\alpha = \frac{-\sqrt{۹۸}}{۱۰}$$

$$-\frac{\sqrt{۲}}{۲}(\cos\alpha+\sin\alpha) = -\frac{\sqrt{۲}}{۲}\left(-\frac{\sqrt{۹۸}}{۱۰}+\frac{\sqrt{۲}}{۱۰}\right) = \frac{۳}{۵}$$