سمیر علایی ۲۸ ۱۵,۲۵

۱ـ

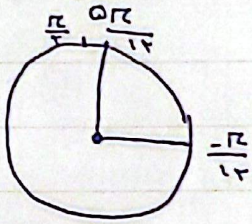
$$\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} = \frac{\cos\alpha}{\sqrt{\sin^2\alpha}} = \frac{\cos\alpha}{|\sin\alpha|} = \frac{\cos\alpha}{\sin\alpha} \Rightarrow \sin\alpha > 0$$

۲

$$\frac{1}{\underbrace{\sqrt{\cos^2\alpha}}_{|\cos\alpha|}} - \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \Rightarrow \cos\alpha > 0$$

ربع اول ✓

۲ـ



$\cancel{\sin 2\alpha = 2\sin x\cos x = \frac{m-1}{4}}$

$\cancel{8\sin x\cos x + 2 = m}$

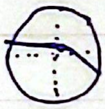
۲

راه دوم و اصلی

$$\sin 2x = \frac{m-1}{4} \qquad -\frac{\pi}{12} < x < \frac{5\pi}{12}$$

$$-\frac{\pi}{6} < 2x < \frac{5\pi}{6} \Rightarrow \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\Downarrow \quad -\frac{1}{2}$$



$$-\frac{1}{2} < \frac{m-1}{4} \le 1$$

$$-2 < m-1 \le 4$$

جواب ← $\boxed{-1 < m \le 5}$ ✓

۳-

$$\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}=\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}=-3 \Rightarrow \boxed{\sin x\cos x=-\frac{1}{3}}$$ ✓

۱٫۵

$$\cot\alpha+\tan\alpha=\frac{2}{\sin 2\alpha}=-3 \Rightarrow$$

$$3\pi<4x<4\pi$$

$$\frac{3}{4}\pi<x<\pi$$

$$-3\sin 2\alpha=2$$

$$\sin 2\alpha=-\frac{2}{3}$$

$$\frac{1}{\sin^3 x+\cos^3 x}=\frac{1}{(\sin x+\cos x)(\sin^2 x+\cos^2 x-\sin x\cos x)}=\frac{1}{(\sin x+\cos x)(1+\frac{1}{3})} \quad (1)$$

$$\frac{\sin^2 x+\cos^2 x+2\sin x\cos x-2\sin x\cos x}{\sin x\cos x}=-3 \Rightarrow$$

$$\frac{(\sin x+\cos x)^2}{\sin x\cos x}-2=-3 \qquad \frac{(\sin x+\cos x)^2}{-\frac{1}{3}}=-1$$

$$(\sin x+\cos x)^2=\frac{1}{3}$$

$$\sin x+\cos x=\frac{1}{\sqrt{3}}=-\frac{\sqrt{3}}{3}$$

(1)

$$\frac{1}{\sin^3 x+\cos^3 x}=\frac{1}{\frac{4}{3}\times\frac{\sqrt{3}}{3}}=\frac{-9}{4\sqrt{3}}$$

$$(\sin\alpha+\cos\alpha)^2=1+2\sin\alpha\cos\alpha$$

$$=1+2(-\frac{1}{3})=\frac{1}{3}$$

$$3\pi<4x<4\pi \rightarrow \frac{3}{4}\pi<x<\pi \xrightarrow{\sin x+\cos x<0} \frac{-\sqrt{3}}{3}$$

$$\sin^3 x+\cos^3 x=(\sin x+\cos x)(\sin^2 x+\cos^2 x-\sin x\cos x)=-\frac{\sqrt{3}}{3}\left(\frac{4}{3}\right)$$

$$\hookrightarrow 1-(-\frac{1}{3})=\frac{4}{3}$$

$$\rightarrow \frac{1}{\sin^3 x+\cos^3 x}=\boxed{\frac{-3}{4}\sqrt{3}}$$

/ /

ع

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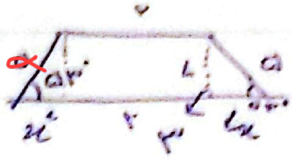
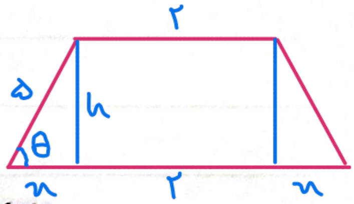


$\cos\theta = ۰/۶ \rightarrow \theta = ۵۳°$

$۳^۲ + x^۲ = ۵^۲ \rightarrow x = ۴$

$\frac{L}{۵} = \frac{۶}{۱۰} \rightarrow L = ۳$

$\frac{(۲ + (۴+۴+۲)) \times ۳}{\ } = ۱۸$



$\cos\theta = \frac{۶}{۱۰} = \frac{x}{۵} \xrightarrow{\ } x = ۳$

$\sin\theta = \frac{۸}{۱۰} = \frac{h}{۵} \rightarrow h = ۴$

$\rightarrow S = \frac{(۲+۸)}{۲} \times ۴ = \boxed{۲۰}$

$\tan(۲۸۵°) = \tan(۱۰۵°)$

$\tan(۱۹۵°) = \tan(۱۵°)$

$\sin(۱۰۹۵)° = \sin(۱۵°)$

$\cos(۲۵۵°) = \cos(۱۰۵°)$

$\cos(۱۹۵°) = -\cos(۱۵°)$

۲ ص

$\frac{\sin ۱۰۵°}{\cos ۱۰۵°} \times \frac{\sin ۱۵°}{\cos ۱۵°} - \sin ۱۵° \times \cos ۱۵° =$

$\frac{\cos ۱۵°}{\cos ۱۰۵°} \times \frac{-\cos ۱۰۵°}{\cos ۱۵°} = -۱$

۴

$\sin ۱۵° = -\cos ۱۰۵°$

$\sin ۱۰۵° = \cos ۱۵°$

$-۱-$

$\underbrace{-\cot ۱۵° \times \tan ۱۵°}_{-۱} - (\sin ۱۵° \times -\sin ۱۵°) = -۱ + \sin^۲ ۱۵° = -\cos^۲ ۱۵°$

$\boxed{K = -۱}$

$\sqrt{3} \times \frac{-\sqrt{3}}{2} \times -\cos 37^\circ = \sqrt{3} \times \frac{\sqrt{3}}{2} \times -\cos(37^\circ) =$ -۶

۲

$\frac{5}{2} \cos 37^\circ \Rightarrow$ برابر $\frac{5}{2}$

دقت!

۱٫۵ -۷

$16 \times \cos^2 \frac{\pi}{12} \times \cos^2 \frac{\pi}{6} \times \cos^2 \frac{\pi}{3} = 16 \times \frac{1+\frac{\sqrt{3}}{2}}{2} \times \left(\frac{\sqrt{3}}{2}\right)^2 \times \frac{1}{4} \cdot \frac{1}{4}$

$\frac{2+\sqrt{3}}{4} \times \sqrt{3} = \boxed{\frac{2\sqrt{3}+3}{4}}$

ربع سوم!

$1-\sin x = 4+4\sin x \Rightarrow \sin x = -\frac{3}{5} \ , \ \cos x = \frac{-4}{5} \Rightarrow$ -۸

$\boxed{323^\circ}$

۰٫۷۵

$\tan^2 \frac{x}{2} = \frac{1-\cos x}{1+\cos x} \Rightarrow \tan^2 \frac{x}{2} = \frac{\frac{1}{5}}{\frac{9}{5}} = \frac{1}{9} \Rightarrow \tan \frac{x}{2} = \cancel{\frac{1}{3}}$

مقدار صحیح!

$\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\cot\frac{\theta}{2} \Rightarrow k=2$

$\cot \frac{\theta}{2}$ $\quad$ $\cot \frac{\theta}{2}$

۲ -۹

$\cos\left(\frac{11\pi}{4} + \alpha\right) = \cos\left(2\pi + \frac{\pi}{2} + \frac{\pi}{4} + \alpha\right) = \sin\left(\frac{\pi}{4} + \alpha\right)$ -۱۰

۱٫۵

$\sin\left(\frac{\pi}{4} + \alpha\right) = \frac{\sin\alpha + \cos\alpha}{-\sqrt{2}} = \frac{\frac{\sqrt{2}}{10} \cdot \frac{-\sqrt{98}}{10} \cdot 7}{-\sqrt{2}} = \frac{8}{10} = \cancel{\alpha}$

$\boxed{0{,}6}$

$\frac{4}{5}$

ناحیه دوم! $\alpha \leftarrow \cos\alpha < 0$

-۷

$$f\left(\frac{\pi}{24}\right) = 16\cos^2\left(\frac{\pi}{12}\right)\cos^2\left(\frac{\pi}{4}\right)\cos^2\left(\frac{\pi}{3}\right)\cos^2\left(\frac{2\pi}{3}\right)$$

$$\cos^2\frac{\pi}{12} = \frac{1+\cancel{\cos\frac{\pi}{6}}^{\frac{\sqrt{3}}{2}}}{2} \rightarrow \cos^2\frac{\pi}{12} = \frac{2+\sqrt{3}}{4}$$

$$f\left(\frac{\pi}{24}\right) = 16\left(\frac{2+\sqrt{3}}{4}\right)\times\frac{3}{4}\times\frac{1}{4}\times\frac{1}{4} = \boxed{\frac{3(2+\sqrt{3})}{16}}$$

-۸

$$\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{-3}{5} \rightarrow 10\tan\frac{x}{2} = -3-3\tan^2\frac{x}{2}$$

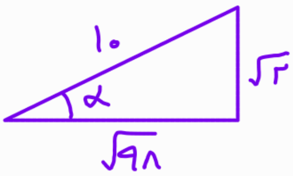
$$\tan\frac{x}{2} = \frac{-1}{3} \times$$ غیر صحیح!

$$\boxed{\tan\frac{x}{2} = -3} \checkmark$$

-۱۰

$$\cos\left(\frac{11\pi}{4}+\alpha\right) = \cos\left(3\pi-\frac{\pi}{4}+\alpha\right) = -\cos\left(\alpha-\frac{\pi}{4}\right)$$

$$= -\left(\cos\alpha\cos\frac{\pi}{4}+\sin\alpha\sin\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}\left(\cos\alpha+\sin\alpha\right)$$



$$\xrightarrow{\text{ناحیه دوم}} \cos\alpha = \frac{-\sqrt{98}}{10}$$

$$-\frac{\sqrt{2}}{2}\left(\cos\alpha+\sin\alpha\right) = -\frac{\sqrt{2}}{2}\left(-\frac{\sqrt{98}}{10}+\frac{\sqrt{2}}{10}\right) = \frac{3}{5}$$