(1

$$\cot_a = \frac{\cos a}{\sqrt{1-\cos^2_a}} \xrightarrow{\sin^2_a = 1-\cos^2_a} \cot_a = \frac{\cos a}{|\sin a|}$$

باید علامت مثبت بیرون بیاید → $\sin a > 0$

$$\frac{1}{\sqrt{\cos^2_a}} - \frac{1}{\cot a} \Longrightarrow \frac{1}{|\cos a|} - \frac{\sin a}{\cos_a} = \frac{1-\sin a}{|\cos a|} \longrightarrow \frac{\sin a}{\cos a} = \frac{\sin a}{|\cos a|}$$

$\cos_a > 0$

$\sin a > 0$, $\cos a > 0 \Longrightarrow$ a در ناحیه اول

(2

$$-\frac{\pi}{12} < x < \frac{5\pi}{12} \xrightarrow{\times 2} -\frac{\pi}{6} < 2x < \frac{5\pi}{6} \xrightarrow{\text{جایگذاری در } \sin} \sin -\frac{\pi}{6} < \sin 2x < \sin\frac{5\pi}{6}$$

max = 1   Min = $-\frac{1}{2}$ $\frac{1}{2}$

$$\xrightarrow{\sin 2x = \frac{m-1}{2}} -\frac{1}{2} < \frac{m-1}{2} \leq 1 \rightsquigarrow -1 < m \leq 3$$

(3

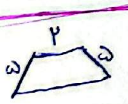
$$\tan_x + \cot_x = \frac{\sin}{\cos} + \frac{\cos}{\sin} = \frac{1}{\sin\cos} = -3 \rightarrow \sin\cos = -\frac{1}{3} \qquad (\sin+\cos)^2 = 1 + 2\sin\cos$$

$$(\sin+\cos)^2 = \frac{1}{3} \xrightarrow{\text{جذر}} |\sin+\cos| = \sqrt{\frac{1}{3}} = \frac{\sqrt{3}}{3} \Longrightarrow \sin_x + \cos_x = -\frac{\sqrt{3}}{3}$$

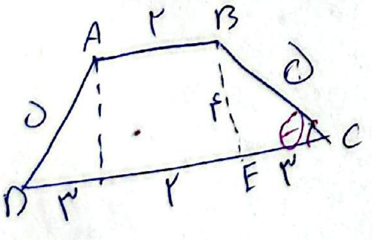
باید منفی باشد طبق $3\pi < 4x < 4\pi$

$$\frac{1}{\sin^3_x + \cos^3_x} \xrightarrow{\text{فرمول چاق و لاغر}} \frac{1}{\sin+\cos} \times \frac{1}{(\sin^2+\cos^2-\sin\cos)} = \frac{1}{\left(\frac{-\sqrt{3}}{3}\right)\left(\frac{4}{3}\right)} = \boxed{-\frac{9}{4\sqrt{3}}}$$

(4



$\cos_\theta = 0{,}6 \rightarrow CE = 3$ فیثاغورس $\rightarrow BE = 4$



$$S = \frac{(DC + AB)(BE)}{2} = \frac{10 \times 4}{2} = 20$$

(5

$$\tan\left(\frac{3\pi}{2}+15\right) \times \tan(15-\pi) - \sin(4\pi+15) \times \cos\left(\frac{3\pi}{2}-15\right) \longrightarrow (-\cot_{15})(\tan_{15}) - (\sin_{15}) \times$$

$$(-\sin_{15}) = -1 - (-\sin^2_{15}) = -\cos^2_{15}$$

$$\frac{-\cos^2_{15}}{K\cos^2_{15}} \Longrightarrow \boxed{K = -1}$$

۶)

$\sqrt{3}\left(-\frac{\sqrt{3}}{2}\right)\left(\sin\left(\frac{3\pi}{2}-27\right)\right)-\sqrt{2}\left(\frac{\sqrt{2}}{2}\right)\cos(\pi-27)=-\frac{3}{2}\left(-\cos_{27^\circ}\right)-\left(-\cos_{27}\right)$

$=2{,}5\cos_{27^\circ}$ → ۲٫۵ برابر

۷)

$f\left(\frac{\pi}{36}\right)=16\cos^2(15)\cos^2(30)\cos^2(60)\cos^2(120)$

$\cos_{15}=\cos(45-30)=\cos_{45}\cos_{30}+\sin_{45}\sin_{30}=\boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

جایگذاری → $16\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)^2\left(\frac{\sqrt{3}}{2}\right)^2\left(\frac{1}{2}\right)^2\left(-\frac{1}{2}\right)^2=\boxed{\frac{6+3\sqrt{3}}{16}}$

۸)

$\frac{1-\sin\alpha}{1+\sin\alpha}=4 \rightarrow 4+4\sin x=1-\sin x \rightarrow \boxed{\sin x=-\frac{3}{5}}\ ,\ \boxed{\cos x=-\frac{4}{5}}$

$\tan\frac{x}{2}=\frac{1-\cos x}{\sin x}=\frac{\sin x}{1+\cos x}=-3$

۹)

$\frac{\sin\alpha}{1+\cos\alpha}=\frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}}=\tan\frac{\alpha}{2}$

$\frac{1-\cos\alpha}{\sin\alpha}=\frac{2\sin^2\frac{\alpha}{2}}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}=\tan\frac{\alpha}{2}$ → $\frac{\sin\alpha}{1-\cos\alpha}+\frac{1+\cos\alpha}{\sin\alpha}=\frac{2\cot\frac{\alpha}{2}}{K\cos\frac{\alpha}{2}}$ → $\boxed{K=2}$

۱۰)

$\cos^2_\alpha=1-\sin^2_\alpha \rightarrow \cos^2_\alpha=1-\frac{2}{10} \rightarrow \cos\alpha=(-)\frac{\sqrt{98}}{10}$ (ربع دوم)

$\rightarrow \cos\left(\frac{11\pi}{4}+\alpha\right)=\left(\cos\frac{11\pi}{4}\right)(\cos\alpha)-\left(\sin\frac{11\pi}{4}\right)(\sin\alpha)=\left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{98}}{10}\right)-\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{10}\right)$

$=\frac{14}{20}-\frac{2}{20}=\boxed{\frac{12}{20}}$