حد توابع متغیری

۱) $\lim_{x\to 1} \frac{4x^2-7x+3}{5x^2-8x+3} = \frac{0}{0} \xrightarrow{\text{هوپ}} \frac{8x-7}{10x-8} = \frac{1}{2}$

۲) $\lim_{x\to 0} \frac{|2x-1|+|2x+1|}{x} = \frac{1-2x-2x-1}{x} = \frac{-4x}{x} \neq \frac{0}{0} \xrightarrow{\text{هوپ}} \frac{-4}{1} = -4$

۳) $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} \xrightarrow{\text{هوپ}} \frac{1}{\frac{1}{2\sqrt{x}}} = 4$

۴) $\lim_{x\to 2} \frac{x-\sqrt{2x}}{2x^2-x-6} = \frac{0}{0} \xrightarrow{\text{هوپ}} \frac{1-\frac{2}{2\sqrt{2x}}}{4x-1} = \frac{\frac{1}{2}}{7} = \frac{1}{14}$

۵) $\lim_{x\to 1} \frac{1-\sqrt{x}}{2-\sqrt{5-x}} = \frac{0}{0} \xrightarrow{\text{هوپ}} \frac{\frac{-1}{2\sqrt{x}}}{\frac{-1}{2\sqrt{5-x}}} = \frac{-\frac{1}{2}}{\frac{1}{4}} = -2$

۶) $\lim_{x\to 4} \frac{\sqrt{2x+4}-4}{\sqrt[3]{5x+7}-3} = \frac{0}{0} \xrightarrow{\text{هوپ}} \frac{\frac{2}{2\sqrt{2x+4}}}{\frac{5}{3\sqrt[3]{(5x+7)^2}}} = \frac{\frac{3}{8}}{\frac{5}{27}} = \frac{81}{40}$

۷) $\lim_{x\to 1} \frac{\sqrt{x^2+3x}-2}{\sqrt[3]{x}-1} = \frac{0}{0} \xrightarrow{\text{هوپ}} \frac{\frac{2x+3}{2\sqrt{x^2+3x}}}{\frac{1}{3\sqrt[3]{x^2}}} = \frac{\frac{5}{4}}{\frac{1}{3}} = \frac{15}{4}$

۸) $\lim_{x\to \pi} \frac{1+\cos^3 x}{\sin^2 x} = \frac{0}{0} \xrightarrow{\text{هوپ}} \frac{3\cos^2 x(-\sin x)}{2\sin x\cos x} = \frac{-3\cos x}{2} = \frac{3}{2}$

۹) $\lim_{x\to \frac{\pi}{4}} \frac{1-\tan x}{\sin x-\cos x} = \frac{0}{0} \xrightarrow{\text{هوپ}} \frac{-\frac{1}{\cos^2 x}}{\cos x+\sin x} = \frac{-1}{\cos x(\cos x+\sin x)} = \frac{-1}{\frac{1}{2}} = -2$

۱۰) $\lim_{x\to \frac{\pi}{4}} \frac{\tan^2 x-1}{\cos 2x} = \frac{0}{0} \xrightarrow{\text{هوپ}} \frac{2\tan x(1+\tan^2 x)}{-2\sin 2x} = \frac{2\tan x\sec^2 x}{-2\sin 2x} = \frac{-1}{\cos^2 x\sin 2x} = \frac{-1}{\frac{1}{2}} = -2$