

الف) $f(x) = \sqrt{\frac{x-1}{x}} - \frac{x}{x-1} \rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{x^2 - 2x + 1 - x^2}{x(x-1)} \geq 0 \Rightarrow \frac{1-2x}{x(x-1)} \leq 0$ (۲)

$\frac{1}{x-1} - \frac{2}{x} + \frac{1}{x-1} \rightarrow Df = (-\infty, 0) \cup [\frac{1}{2}, +\infty)$ ✓

ب) $f(x) = \frac{\frac{1}{x-1} - \frac{2}{x}}{\frac{2}{x-1} + \frac{1}{x+2}} = \frac{\frac{x-2x+2}{x(x-1)}}{\frac{2x+4x-1}{(x-1)(x+2)}} = \frac{-x+2}{x(x+2)} \rightarrow Df = \mathbb{R} - \left\{ -2, -1, 0, 1, \frac{2}{3} \right\}$ ✓

الف) $f(x) = \sqrt{\left(\frac{x}{1+x} - 9\right)(22-x^2)} \rightarrow \left(\frac{x}{1+x} - 9\right)(22-x^2) \geq 0$ (۲)

$Df = (-\infty, -2] \cup \left[\frac{0}{2}, +\infty\right)$ ✓

ب) $\sqrt{x-1} + \sqrt{y+1} = 3 \Rightarrow \sqrt{y+1} = 3 - \sqrt{x-1} \Rightarrow \begin{cases} (I) x-1 \geq 0 \Rightarrow x \geq 1 \\ (II) 3 - \sqrt{x-1} \geq 0 \Rightarrow \sqrt{x-1} \leq 3 \end{cases}$

$\sqrt{x-1} \leq 3 \xRightarrow{(I)} x-1 \leq 9 \Rightarrow x \leq 10 \Rightarrow Df = [1, 10]$ ✓

$f(x) = \frac{\log(x^2 - x + 2)}{\sqrt{x^2 - 1} + 1} \Rightarrow \begin{cases} (I) x^2 - x + 2 > 0 \Rightarrow (x+1)(x-2) > 0 \Rightarrow \frac{1}{x+1} - \frac{2}{x-2} \rightarrow (-\infty, -1) \cup (2, +\infty) \\ (II) x^2 - 1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow \frac{x}{x-1} \Rightarrow (-\infty, -1] \cup [1, +\infty) \\ (III) \sqrt{x^2 - 1} + 1 \neq 0 \Rightarrow \sqrt{x^2 - 1} \neq -1 \end{cases}$

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$\Rightarrow Df = (-\infty, -1) \cup (2, +\infty)$ ✓ (۲)

$\sqrt{2+ax-x^2} \Rightarrow Df = [-2, b]$ (۲)

$-x^2 + ax + 2 \geq 0 \Rightarrow b, -2 \text{ از آن } = 0$

$x = -2 \Rightarrow -4 - 2a + 2 = 0 \Rightarrow -2a = 2 \Rightarrow a = -1$

$x = b \Rightarrow -b^2 - \frac{1}{4}b + 2 = 0 \Rightarrow b = \frac{3}{4}$

$a+b = -\frac{1}{4} + \frac{3}{4} = \frac{2}{4} = \frac{1}{2}$ ✓

$g(x) = \sqrt{f(x) - x} \Rightarrow f(x) - x \geq 0$

$f(x) = \begin{cases} 2x-2 & x \geq 1 \\ 2x+2 & x < 1 \end{cases} \Rightarrow f(x) - x = \begin{cases} x-2 & x \geq 1 \\ x+2 & x < 1 \end{cases} \Rightarrow \begin{cases} x-2 \geq 0 & x \geq 1 \Rightarrow x \geq 1 \\ x+2 \geq 0 & x < 1 \Rightarrow x \geq -2 \end{cases}$ ✓

$Df = [-2, +\infty)$ ✓ (۲)

$$f(x) = \begin{cases} (a+1)(x+2) & x > 1 \\ 3a+2x & x \leq 1 \end{cases} \quad \text{و } f(1) = f(-2) + a$$

$$f(1) = f(-2) + a \Rightarrow 2 \times ((a+1) \cdot 1) = 3a - 4 + a \Rightarrow 14a + 14 = 4a - 4$$

$$10a = -18 \Rightarrow a = -1.8 \checkmark$$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 \quad f(2-\sqrt{3}) + f(2+\sqrt{3})$$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 \quad (2+\sqrt{3}) \times (2-\sqrt{3}) = 1 \Rightarrow \text{مقلوب، مقلوب، مقلوب، مقلوب، مقلوب، مقلوب}$$

$$f(2-\sqrt{3}) + f(2+\sqrt{3}) = 2f(2+\sqrt{3})$$

$$f(2+\sqrt{3}) = \sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 2 = \sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}} + 2 = \sqrt{4} + 2 = 4$$

$$2f(2+\sqrt{3}) = 8 \checkmark$$

$$\begin{cases} 2f(x) - 3f(-x) = 4x^2 - x \\ 2f(-x) - 3f(x) = 4x^2 + x \end{cases} \rightarrow \begin{cases} 4f(x) - 9f(-x) = 14x^2 - 2x \\ 9f(-x) - 4f(x) = 14x^2 + 2x \end{cases}$$

$$-5f(x) = 20x^2 + 2x$$

$$f(x) = -4x^2 - \frac{1}{5}x \checkmark$$

$$(x+2)f(x) - 3xf(x+2) = 4x^2 - mx + 3m - 1$$

$$x=0 \Rightarrow 4f(2) = 14 + 2m + 3m - 1 = 5m + 13$$

$$x=2 \Rightarrow 2f(2) = 3m - 1 \Rightarrow 4f(2) = 9m - 2 \Rightarrow 5m + 13 = 9m - 2 \Rightarrow 4m = 15 \Rightarrow m = 3.75$$

$$\Rightarrow 4f(2) = 22.5 + 13 = 35.5$$

$$f(2) = 8.875 \checkmark$$

$$x=-1 \Rightarrow f(-1) + f(-1) = \frac{3(1) - 12(1) + 3}{-1}$$

$$\Rightarrow 2f(-1) = \frac{14}{-1} = -14 \Rightarrow f(-1) = -7 \checkmark$$