

الف) $f(x) = \sqrt{\frac{x+1}{x}} - \frac{x}{x-1}$ $\frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{-2x+1}{x^2-x} \geq 0$ $D_f = (-\infty, 0) \cup [\frac{1}{2}, +\infty)$

$\frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{x^2-x-x^2+x}{x^2-x} \geq 0 \rightarrow \frac{-2x+1}{x^2-x} \geq 0$

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ب) $f(x) = \frac{1}{x+2} - \frac{2}{x}$ $\frac{x-2x-4}{x^2+2x} \rightarrow \frac{x^2+x-4}{(x+4)(x^2+2x)}$ $R = \{-\frac{4}{3}, -2, 0, 1\}$

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الف) $f(x) = \sqrt{(\frac{1}{x})^2 - 9} (2x - \frac{1}{x})$ $\frac{1}{x^2} - 9 = 0 \rightarrow \frac{1}{x^2} = 9 \rightarrow x = \pm \frac{1}{3}$ $x = -\frac{1}{3}$ $D_f = (-\infty, -\frac{1}{3}] \cup [\frac{1}{3}, +\infty)$

$\sqrt{y+1} = x - \sqrt{x-1} \rightarrow x - \sqrt{x-1} \geq 0 \rightarrow x-1 \geq 0 \rightarrow x \geq 1$

$x^2 \geq x-1 \rightarrow x^2 - x + 1 \geq 0 \rightarrow x \geq 1$

$D_f = [1, 2]$

$f(x) = \frac{\log(x^2-x-2)}{\sqrt{x^2-1}+1}$ $x^2-x-2 > 0 \rightarrow (x-2)(x+1) > 0 \rightarrow x < -1 \text{ or } x > 2$

$x^2-1 \geq 0 \rightarrow x^2 \geq 1 \rightarrow x \geq 1 \text{ or } x \leq -1$

$D_f = (2, +\infty) \cup (-\infty, -1)$

$\sqrt{x - \frac{1}{x}} - x^2$ $-x - 2x + x^2 = 0 \rightarrow x^2 - 3x = 0 \rightarrow x(x-3) = 0 \rightarrow x = 0 \text{ or } x = 3$

$\frac{1}{x} + 1 = 1, 2, 3$ $\frac{1}{x} + 1 = 1 \rightarrow \frac{1}{x} = 0 \rightarrow x = \infty$ $\frac{1}{x} + 1 = 2 \rightarrow \frac{1}{x} = 1 \rightarrow x = 1$ $\frac{1}{x} + 1 = 3 \rightarrow \frac{1}{x} = 2 \rightarrow x = \frac{1}{2}$

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$f(x) = x \geq 0$ $f(x) \geq x$ $2x-2; x \geq 1 \rightarrow 2x-2-x \geq 0 \rightarrow x-2 \geq 0 \rightarrow x \geq 2$ $2x+2; x < 1 \rightarrow 2x+2-x \geq 0 \rightarrow x+2 \geq 0 \rightarrow x \geq -2$ $-2 \leq x < 1$ $D_f = [-2, +\infty)$

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$$f(-r) = -f + r^4$$

$$f(a) = \sqrt{a} + \sqrt{a}$$

$$r f(a) = f(-r) + a$$

$$1 \sqrt{a} + 1 \sqrt{a} = -f + r^4 + f$$

$$a = -1/1 \quad \checkmark$$

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$$f(x) = \frac{\sqrt{x} + r\sqrt{x} + 1}{\sqrt{x}} = \frac{(\sqrt{x} + 1)^r}{\sqrt{x}} = \sqrt{x} + \frac{1}{\sqrt{x}} + r$$

$$x_1 = r + \sqrt{r}$$

$$x_2 = r - \sqrt{r}$$

$$\sqrt{x} = a$$

$$f(x_1) = a + \frac{1}{a} + r$$

$$f(x_2) = \frac{1}{a} + a + r$$

$$x_1 \times x_2 = r - r = 0$$

$$x_1 = \frac{1}{x_2}$$

$$a^r + \frac{1}{a^r} = r + \sqrt{r} + r - \sqrt{r} = r$$

$$(a + \frac{1}{a})^r = a^r + \frac{1}{a^r} + r = r$$

$$a + \frac{1}{a} = \sqrt{r}$$

$$f(x_1) + f(x_2) = r(a + \frac{1}{a} + r) = r(\sqrt{r} + r)$$

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$$r f(x) - r f(-x) = f x^r - x$$

$$r f(-x) - r f(x) = f x^r + x$$

$$\left. \begin{aligned} r f(x) - r f(-x) &= f x^r - x \\ - r f(x) &= \Lambda x^r - r x + 1 r x^r + r x = r \Lambda x^r + x \end{aligned} \right\}$$

$$- \Lambda f(x) = r \Lambda x^r + x$$

$$f(x) = -f x^r = \frac{x}{\Lambda} \quad \checkmark$$

$$(r) f(0) = 0 = r m - 1$$

$$r f(0) = r m - 1 \rightarrow f(0) = \frac{r m - 1}{r} \quad \textcircled{\text{I}}$$

$$x = -r \rightarrow 0 f(-r) + 4 f(0) = 1 r + r m + r m - 1 = 1 \Lambda + \Lambda m$$

$$4 f(0) = 1 \Lambda + \Lambda m \rightarrow f(0) = \frac{1 \Lambda + \Lambda m}{4} \quad \textcircled{\text{II}}$$

$$f(0) = \frac{(r \times \frac{9}{r}) - 1}{r} = \frac{r \Lambda}{r} \quad \checkmark$$

$$\left. \begin{aligned} \textcircled{\text{I}} \\ \textcircled{\text{II}} \end{aligned} \right\} \begin{aligned} r m - 1 &= 1 \Lambda + \Lambda m \\ r m &= 1 \Lambda \\ m &= \frac{1 \Lambda}{r} \quad \checkmark \end{aligned}$$

$$x = 1 \rightarrow f(1) + f(1) = r - 1 r + r$$

$$r f(1) = -r$$

$$f(1) = -1$$

$$x = -1 \rightarrow f(-1) + f(-1) = \frac{r + 1 r + r}{-1} = -1 \Lambda$$

$$r f(-1) = -1 \Lambda \rightarrow f(-1) = -\frac{1 \Lambda}{r} \quad \checkmark$$

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