نام و نام خانوادگی ...آرشام یاری.... پاسخنامه تشریحی تکلیف شماره ...۴... کلاس ....یازدهم......

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الف) $\sqrt{\dfrac{(x-1)^2-x^2}{x(x-1)}} = \sqrt{\dfrac{1-2x}{x(x-1)}} \rightarrow \dfrac{1-2x}{x(x-1)} \geq 0$

$\begin{array}{ccccccc} & 0 & & \frac{1}{2} & & 1 & \\ + & \circ & - & \bullet & + & \circ & - \end{array}$

$\Rightarrow D_f: (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $\dfrac{\dfrac{-x-2}{x(x+1)}}{\dfrac{4x+5}{x^2+x-2}} \Rightarrow \begin{cases} x(x+1) \neq 0 \\ \Rightarrow x \neq \{0, -1\} \\ 4x+5 \neq 0 \Rightarrow x \neq \{-\frac{5}{4}\} \\ x^2+x-2 \neq 0 \Rightarrow x \neq \{1, -2\} \end{cases} \xrightarrow{\text{اشتراک}} D_f: \mathbb{R} - \{0, -1, 1, -2, -\frac{5}{4}\}$

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الف) $\left((\frac{1}{3})^x - 9\right)(32 - 4^x) \geq 0$

$\begin{array}{ccccc} & -2 & & \frac{5}{2} & \\ + & \bullet & - & \bullet & + \end{array} \Rightarrow D_f: (-\infty, -2] \cup [\frac{5}{2}, +\infty)$

$(\frac{1}{3})^x - 9 = 0 \Rightarrow x = -2 \qquad 32 - 4^x = 0 \Rightarrow x = \frac{5}{2}$

ب) $\sqrt{y+1} = 3 - \sqrt[4]{x-1} \Rightarrow x-1 \geq 0 \Rightarrow x \geq 1$

$\longrightarrow 3 - \sqrt[4]{x-1} \geq 0 \Rightarrow 3 \geq \sqrt[4]{x-1} \Rightarrow 81 \geq x-1$

$\Rightarrow 82 \geq x$

$\Rightarrow D_f: [1, 82]$

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$\log_4 (x^2-x-2) \longrightarrow x^2-x-2 > 0$

$(x-2)(x+1) > 0$

$\begin{array}{ccccc} & -1 & & 2 & \\ + & \circ & - & \circ & + \end{array} \rightarrow (-\infty, -1) \cup (2, +\infty) \quad (I)$

$\sqrt{x^2-1} \rightarrow x^2-1 \geq 0 \Rightarrow x^2 \geq 1 \rightarrow \begin{cases} x \geq 1 \\ x \leq -1 \end{cases} \longrightarrow (-\infty, -1] \cup [1, +\infty) \quad (II)$

$\sqrt{x^2-1} + 1 \neq 0$ همواره برقرار ✓. $\Longrightarrow I \cap II \rightarrow D_f: (-\infty, -1) \cup (2, +\infty)$

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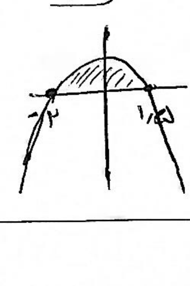
عبارت زیر رادیکال یک سهمی max دار است و در نقاط $\{b\}$ و $\{-2\}$ صفر می‌شود

$3 + ax - x^2 \xrightarrow{x=-2} 3 - 2a - 4 = 0 \Rightarrow -2a - 1 = 0 \rightarrow -2a = 1 \rightarrow a = -\frac{1}{2}$

$\Rightarrow -x^2 - \frac{1}{2}x + 3 \rightarrow x = \dfrac{\frac{1}{2} \pm \sqrt{\frac{1}{4} + 12}}{-2} \Rightarrow x = \{-2, 1.5\}$



$\Rightarrow D_f = [-2, 1.5] \Rightarrow b = 1.5 \Rightarrow a + b = -\frac{1}{2} + 1.5 = 1$

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$x \geq 1 \rightarrow g(x) = \sqrt{3x-2-x} = \sqrt{2x-2} \Rightarrow 2x-2 \geq 0$

$2x \geq 2 \Rightarrow x \geq 1$ ✓

$x < 1 \rightarrow g(x) = \sqrt{2x+3-x} = \sqrt{x+3} \Rightarrow x+3 \geq 0$

$x \geq -3 \Rightarrow -3 \leq x < 1$

اشتراک $\Rightarrow D_f = [-3, +\infty)$

$-2 < 1 \rightarrow f(-2) = 3a - 4$

$5 > 1 \rightarrow 2f(5) = 2(a+1)(5+2) \Rightarrow 2f(5) = 14a + 14$

$2f(5) = f(-2) + a \xrightarrow{f(-2) = 3a-4} 2f(5) = 4a - 4$ ۶

$\Rightarrow 14a + 14 = 4a - 4$

$10a = -18$

$a = -1.8$

می دانیم $2+\sqrt{3}$ و $2-\sqrt{3}$ معکوس یکدیگرند چون $(2-\sqrt{3})(2+\sqrt{3}) = 1$

$2-\sqrt{3} = t \Rightarrow f(2-\sqrt{3}) + f(2+\sqrt{3}) = f(t) + f(\frac{1}{t})$ $\quad f(t) = \sqrt{t} + \frac{1}{\sqrt{t}} + 2$

$\Rightarrow f(t) = f(\frac{1}{t}) \Rightarrow f(t) + f(\frac{1}{t}) = 2f(t)$ $\quad f(\frac{1}{t}) = \sqrt{\frac{1}{t}} + \sqrt{t} + 2$ ۷

$2f(t) = 2\left(\sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} + 2\right) = 2\left(\underbrace{\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}}}_{Q} + 2\right) = 2(\sqrt{6} + 2) = 2\sqrt{6} + 4$

$\frac{1}{\sqrt{2-\sqrt{3}}} = \sqrt{2+\sqrt{3}}$ $\quad Q^2 = 2-\sqrt{3} + 2 + \sqrt{3} + 2\sqrt{4-3} = 6 \Rightarrow Q = \sqrt{6}$

$2f(x) - 3f(-x) = 4x^2 - x \xrightarrow{\times 2} 4f(x) - 6f(-x) = 8x^2 - 2x$

$2f(-x) - 3f(x) = 4x^2 + x \xrightarrow{\times 3} 6f(-x) - 9f(x) = 12x^2 + 3x$ ۸

$\Rightarrow 4f(x) - 6f(-x) + 6f(-x) - 9f(x) = 20x^2 + x$

$-5f(x) = 20x^2 + x \Rightarrow f(x) = \frac{20x^2 + x}{-5} = -4x^2 - \frac{x}{5}$

$x = 0 \rightarrow 2f(0) = 3m - 1 \xrightarrow{\times 3} 6f(0) = 9m - 3$

$x = -2 \rightarrow 6f(0) = 15 + 5m$

$\Rightarrow 9m - 3 = 15 + 5m$

$4m = 18$ ۹

$m = \frac{18}{4}$

$\Rightarrow f(0) = \frac{3m-1}{2} \xrightarrow{m = \frac{18}{4}} f(0) = \frac{12.5}{2} = 6.25$

$x = -1 \rightarrow f(-1) + f(-1) = \frac{3 + 12 + 3}{-1}$

$2f(-1) = -18 \Rightarrow f(-1) = -9$ ۱۰