پرهام مقتدری — ارزشیابی ریاضی سری B — ۲۵

۲

$x \neq 0$

$x-1 \neq 0 \Rightarrow x \neq 1$

$$f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}}$$

$$\frac{(x-1)^2-x^2}{x(x-1)}=\frac{\cancel{x^2}+1-2x-\cancel{x^2}}{x^2-x} \rightarrow \frac{1}{2}$$

	0		$\frac{1}{2}$		1	
$+$		$-$		$+$		$-$

$D_f=(-\infty,\ 0)\cup[\frac{1}{2},\ 1)$

$$f(x)=\frac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}}$$

$x \neq -1,\ 0,\ 1,\ -2$

$4x+5 \rightarrow x \neq -\frac{5}{4}$

$\frac{3}{x-1}+\frac{1}{x+2} \neq 0 \qquad \frac{3x+6+x-1}{(x-1)(x+2)}$

$D_f=R-\{-1,\ 0,\ 1,\ -2,\ -\frac{5}{4}\}$

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$$f(x)=\sqrt{\underbrace{\left(\left(\frac{1}{3}\right)^x-9\right)}_{x=-2}\underbrace{\left(32-4^x\right)}_{x=\frac{5}{2}}}$$

	-2		$\frac{5}{2}$	
$+$		$-$		$+$

$D_f=(-\infty,\ -2]\cup[\frac{5}{2},\ +\infty)$

$\sqrt[4]{x-1}+\sqrt{y+1}=3 \qquad x-1\geq 0 \qquad x\geq 1$

$\sqrt{y+1}=3-\sqrt[4]{x-1} \Rightarrow 3-\sqrt[4]{x-1}\geq 0 \qquad x-1\leq 81$

$D_f=[1\quad 82] \qquad x\leq 82$

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$$f(x)=\frac{\log_4(x^2-x-2)}{\sqrt{x^2-1}+1}$$

$x^2-x-2>0 \qquad -1,\ 2 \qquad -\frac{1}{2}$

	-1		2	
$+$		$-$		$+$

$\sqrt{x^2-1}+1\neq 0$ هیچگاه صفر نمی‌شود

$x^2-1\geq 0$

	-1		1	
$+$		$-$		$+$

$D_f=(-\infty\quad -1)\cup(2\quad +\infty)$

۴-

$\sqrt{3+ax-x^2}$

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$-(a+2)(x-\frac{3}{2}) = -x^2 - \frac{1}{2}x + 3 \Rightarrow a = -\frac{1}{2}$

$\frac{3}{2} + (-\frac{1}{2}) = 1 \qquad b = \frac{3}{2}$

۵-

$f(x) = \begin{cases} 3x-2 & ; x \geq 1 \\ 2x+3 & ; x < 1 \end{cases}$

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$f(x) - x \geq 0 \xrightarrow{x \geq 1} 3x - 2 - x = 2x - 2 \Rightarrow x \geq 1$

$f(x) - x \geq 0 \xrightarrow{x<1} 2x + 3 - x = x + 3 \Rightarrow x \geq -3$

$D_f = [-3 \quad +\infty)$

۶-

$2f(a) = f(-2) + a \qquad f(x) = \begin{cases} (x+1)(x+2) & ; x \geq 1 \\ 3x + 2x & ; x < 1 \end{cases}$

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$|2a+1|^2 = 3a - 4 + a$

$10a = -18$

$a = -1.8$

۷-

$f(2-\sqrt{3}) + f(2+\sqrt{3})$

۲

$\sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} + 2 + \sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 2 = \quad \times \frac{\sqrt{2}}{\sqrt{2}}$

$\frac{\sqrt{4-2\sqrt{3}}}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{4-2\sqrt{3}}} + \frac{\sqrt{4+2\sqrt{3}}}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{4+2\sqrt{3}}} + 4 =$

$\frac{\sqrt{3}-1}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{3}-1} + \frac{\sqrt{3}+1}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{3}+1} + 4 \Rightarrow \frac{2\sqrt{3}}{\sqrt{2}} + \frac{\sqrt{2}(\sqrt{3}-1)}{2} + \frac{\sqrt{2}(\sqrt{3}+1)}{2} + 4$

$$\frac{4\sqrt{3}}{\sqrt{2}} + 4 = 2\sqrt{6} + 4$$

-8

$2f(x) - 3f(-x) = 4x^2 - x$

$x = -x \rightarrow$ $2f(-x) - 3f(x) = 4x^2 + x$

$4f(x) - 6f(-x) = 8x^2 - 2x$

$6f(-x) - 9f(x) = 12x^2 + 3x$

$-5f(x) = 20x^2 + x$

$f(x) = \frac{20x^2 + x}{-5} = -4x^2 - \frac{x}{5}$

2

-9

$(x+2)f(x) - 3xf(x+2) = 4x^2 - mx + 3m - 1$

$\xrightarrow{f(0)}$ $2(f(0)) - 0 = 3m - 1$ $\qquad 2f(0) = 3m - 1 \xrightarrow{\times 3} 6f(0) = 9m - 3$

$\xrightarrow{f(-2)}$ $0 + 6f(0) = 16 + 2m + 3m - 1$ $\qquad 6f(0) = 5m + 15$

$9m - 3 = 5m + 15$

$6.25 = f(0) = \frac{25}{4} \Longleftarrow$ $\qquad 4m = 18$, $m = 4.5$

2

-10

$f(x) + f(\frac{1}{x}) = \frac{3x^2 - 12x + 3}{x}$

$= ax + b + \frac{a}{x} + b = 3x + \frac{3}{x} - 12$

$a = 3$

$2b = -12$

$b = -6$

2

$\cancel{2f(1) = -6}$

$\cancel{f(1) = -3}$

$f(x) = 3x - 6 \quad \xrightarrow{f(-1)} \quad f(-1) = -3 - 6 = -9$