۱- الف)

$$\frac{x-1}{x} \geq \frac{x}{x-1} \Rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0$$

مخرج مشترک ←

$$\frac{x^2-2x+1-x^2}{x^2-x} \Rightarrow \frac{-2x+1}{x^2-x} \geq 0$$

$x \neq 0$
$x \neq 1$ (۱)

$x_1 = \frac{1}{2}$ $\quad x_2 = 0$ $\quad x_3 = 1$

	$(-\infty, 0)$	0	$(0, \frac{1}{2}]$	$\frac{1}{2}$	$(\frac{1}{2}, 1)$
$-2x+1$	+	+	+	0	−
x^2-x	+	0	−	0	+
$\frac{-2x+1}{x^2-x}$	+	∄	−	0	+

(۱)

$D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب)

$x-1 \neq 0$ $\quad x+1 \neq 0$ $\quad \frac{3}{x-1} + \frac{1}{x+2} \neq 0$

$x+2 \neq 0$ $\quad x \neq 0$

$\frac{3}{x-1} \neq \frac{1}{-2-x} \neq \frac{3}{-3-3x}$

$-1-x = 3+3x$
$4x = -4$
$x = -1$

$x_1 = 1$ $\quad x_2 = -1$ $\quad x_3 = -2$ $\quad x_4 = 0$

$x_5 = -\frac{3}{5}$

$D_f = R - \{1, -1, -2, 0, -\frac{3}{5}\}$ ← $-\frac{3}{5}$

۲- الف)

$3^{2x} - 4^x = 0$
$3^{2x} = 4^x$
$(\frac{1}{3})^x - 9 = 0$
$(\frac{1}{3})^x = 9$
$x_2 = -2$

$2x = 5$
$x_1 = \frac{5}{2}$

	$(-\infty, -2)$	-2	$(-2, \frac{5}{2})$	$\frac{5}{2}$	$(\frac{5}{2}, +\infty)$
$3^{2x} - 4^x$	+	0	−	0	+

$D_f = (-\infty, -2] \cup [\frac{5}{2}, +\infty)$

ب)

$\sqrt{y+1} = 3 - \sqrt[4]{x-1} \Rightarrow 3 - \sqrt[4]{x-1} \geq 0$

$3 \geq \sqrt[4]{x-1}$

$x-1 \geq 0$
$x \geq 1$ (۱)

$81 \geq x-1$
$82 \geq x$ (۲)

$D_f = [1, 82]$

$[1, 82]$

۳-

$$f(x)=\frac{\log_{\frac{1}{2}}(x^2-x-2)}{\sqrt{x^2-1}+1}$$

$x^2-1\geq 0$

$x^2\geq 1$

$-1\geq x$

$x\geq 1$ (۱)

$\sqrt{x^2-1}+1\neq 0$

$\sqrt{x^2-1}\neq -1$

همواره برقرار است. OR (۲)

$x^2-x-2>0$

$(x-2)(x+1)>0$ $\quad x_1=2 \quad x_2=-1$

-1	2
+	−	+

$(-\infty,-1)\cup(2,+\infty)$ (۳)

(۱) ∩ (۲) ∩ (۳) : $(-\infty,-1)\cup(2,+\infty)$

۴-

$-x^2+ax+2\geq 0$ $\qquad a+b=$

$x_1=-2 \qquad x_2=b$

$-4-2a+2=0$ $\qquad -x^2+\frac{1}{4}x+2$

$-1=2a$ $\qquad x_1\times x_2=\frac{c}{a}=-2 \Longrightarrow -2\times x_2=-2$

$a=-\frac{1}{2}$ $\qquad b=x_2=\frac{2}{2}$

$b=\frac{2}{2} \qquad a=-\frac{1}{2}$ $\qquad \boxed{a+b=1}$

۵-

$f(x)-x\geq 0$

$f(x)\geq x$

-2	1
− 0	+	0	+

$3x-2 \;;\; x\geq 1 \Longrightarrow 2x-2-x\geq 0 \Rightarrow 2x-2\geq 0$

$x\geq 1$

$2x+2 \;;\; x<1 \Rightarrow 2x+2-x\geq 0$

$x+2\geq 0$

$x\geq -2$

$-2\leq x<1$

$D_g=[-2,+\infty)$

Date No

۶-

$2f(\omega) = f(-2) + a$

$f(-2) = -\varepsilon + 3a$

$f(\omega) = 7a + 7$

$14a + 14 = -\varepsilon + 3a + a$

$10a = -18$

$a = -1,8$

۷-

$f(x) = \frac{\sqrt{x}^2 + 2\sqrt{x} + 1}{\sqrt{x}} = \frac{(\sqrt{x}+1)^2}{\sqrt{x}} = \sqrt{x} + \frac{1}{\sqrt{x}} + 2$

$x_1 = 2 + \sqrt{3}$

$x_2 = 2 - \sqrt{3}$

$x_1 \times x_2 = 4 - 3 = 1$

$t^2 + \frac{1}{t^2} = 2\sqrt{3} + 2 - 2\sqrt{3} = 4$

$(t + \frac{1}{t})^2 = t^2 + \frac{1}{t^2} + 2 = 4 + 2 = 6 \Rightarrow$

$t + \frac{1}{t} = \sqrt{6}$

$\sqrt{x_1} = t$

$f(x_1) = t + \frac{1}{t} + 2$

$f(x_2) = \frac{1}{t} + t + 2$

$f(x_1) + f(x_2) = 2(t + \frac{1}{t} + 2) \Rightarrow f(x_1) + f(x_2) = 4 + 2\sqrt{6}$

۸-

$2f(x) - 3f(-x) = 4x^2 - x$ (1)

$\times 2$

$2f(-x) - 3f(x) = 4x^2 + x$ (2)

$\times 3$

$\Rightarrow 4f(x) - 4f(x) - 9f(x) + 6f(x) =$

$-5f(x) = 8x^2 - 2x + 12x^2 + 3x = 20x^2 + x$

$-5f(x) = 20x^2 + x$

$f(x) = -4x^2 - \frac{x}{5}$

.9

$f(x+1)f(x) - 0 = 3m-1$

$f(x) = \frac{3m-1}{2} = \frac{9m-3}{6}$ ①

$x = -2 \Rightarrow 0 f(-2) + 4 f(x) = 16 + 2m + 3m - 1 : 15 + 5m$

$4 f(x) = 15 + 5m$

$f(x) = \frac{15+5m}{4}$ ②

①, ② $\Rightarrow 9m - 3 = 15 + 5m$

$4m = 18$

$m = \frac{18}{4}$

$f(x) = \frac{(3 \times \frac{9}{2}) - 1}{2} = \frac{25}{4}$

$\frac{18}{4}$

.10

$f(x)$

$x = 1 \Rightarrow f(1) + f(1) = 3 - 12 + 3$

$2f(1) = -6$

$f(1) = -3$

$x = -1 \Rightarrow f(1) + f(-1) = \frac{3 + 12 + 3}{-1} = -18$

$2f(-1) = -18$

$f(-1) = -9$