B یازدهم تیر ۱۹,۷۵

تکلیف ۴

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۲,۷۵

الف ۱) $f(x) = \sqrt{-\frac{x-1}{x} - \frac{x}{x-1}} \rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0$

$\rightarrow \frac{x^2 - 2x + 1 - x^2}{x(x-1)} \geq 0 \rightarrow \frac{2x-1}{x(x-1)} \leq 0$

دقت!

$\rightarrow$ (sign table: 0, $\frac{1}{2}$, 1: $- \; + \; - \; +$) $\rightarrow D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب ۱) $f(x) = \dfrac{\frac{1}{x+1} - \frac{2}{x}}{\frac{3}{x-1} + \frac{1}{x+2}} = \dfrac{\frac{x-2x-2}{x(x+1)}}{\frac{3x+6+x-1}{(x-1)(x+2)}} = \dfrac{\frac{-x-2}{x(x+1)}}{\frac{4x-5}{(x-1)(x+2)}}$

$\rightarrow D_f = \mathbb{R} - \{1, 0, -1, -2, -\frac{5}{4}\}$ ✓

الف ۲) $f(x) = \sqrt{\left((\frac{1}{3})^x - 9\right)(32 - 4^x)} \rightarrow \left((\frac{1}{3})^x - 9\right)(32 - 4^x) \geq 0$

$\rightarrow$ (sign table: -2, $\frac{5}{2}$: $+ \; - \; +$) $\rightarrow D_f = (-\infty, -2] \cup [\frac{5}{2}, +\infty)$ ✓

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ب ۲) $\sqrt[4]{x-1} + \sqrt{y+1} = 3 \rightarrow \sqrt{y+1} = 3 - \sqrt[4]{x-1}$

$\rightarrow \begin{cases} \text{I} & x - 1 \geq 0 \rightarrow x \geq 1 \\ \text{II} & \sqrt[4]{x-1} \leq 3 \rightarrow x - 1 \leq 81 \rightarrow x \leq 82 \end{cases} \rightarrow D_f = [1, 82]$ ✓

۳) $f(x) = \dfrac{\log_4 (x^2 - x - 2)}{\sqrt{x^2-1} + 1} \rightarrow$

I) $x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0 \rightarrow$ (sign table: -1, 2: $+ \; - \; +$) $\rightarrow (-\infty, -1) \cup (2, +\infty)$

II) $x^2 - 1 \geq 0 \rightarrow x^2 \geq 1 \rightarrow x \geq 1,\; x \leq -1 \rightarrow (-\infty, -1] \cup [1, +\infty)$

III) $\sqrt{x^2-1} + 1 \neq 0 \rightarrow \sqrt{x^2-1} \neq -1 \rightarrow \mathbb{R}$

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$\text{I} \cap \text{II} \cap \text{III}$: $D_f = (-\infty, -1) \cup (2, +\infty)$ ✓

۴) $\sqrt{3 + ax - x^2} \rightarrow D_f = [-2, b]$

$\rightarrow -x^2 + ax + 3 \geq 0$ بازای $\frac{}{b \quad -2} \geq 0$

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$\rightarrow x = -2 \rightarrow -4 - 2a + 3 = 0 \rightarrow -2a = 1 \rightarrow a = -\frac{1}{2}$ ✓

$\rightarrow x = b \rightarrow -b^2 + ab + 3 = 0 \rightarrow -b^2 - \frac{b}{2} + 3 = 0$

$\rightarrow b = \frac{3}{2} \rightarrow a + b = -\frac{1}{2} + \frac{3}{2} = 1$ ✓

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۵) $g(x)=\sqrt{f(x)-x} \rightarrow f(x)-x \geq 0$

$f(x)=\begin{cases} 3x-2 \ , & x\geq 1 \\ 2x+3 \ , & x<1 \end{cases} \rightarrow f(x)-x=\begin{cases} 2x-2 \ , & x\geq 1 \rightarrow x\geq 1 \\ x+3 \ , & x<1 \rightarrow x\geq -3 \end{cases}$

$\rightarrow D_f=[-3 \ , +\infty)$

۶) $f(x)=\begin{cases} (a+1)(x+2) \ , & x>1 \\ 3a+2x & x\leq 1 \end{cases}$ $\quad 2f(5)=f(-2)+a$

$\rightarrow 2f(5)=f(-2)+a \rightarrow 2(a+1)(7)=3a+(-4)+a$

$\rightarrow 14a+14=4a-4 \rightarrow 10a=-18 \rightarrow a=\frac{-18}{10}$

۷) $f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2$ $\quad f(\sqrt{3}+2)+f(2-\sqrt{3})$

$\Downarrow$

$f(x)=\sqrt{x+\frac{1}{x}+2}+2$ $\quad (2+\sqrt{3})(2-\sqrt{3})=1 \rightarrow$ چون ۲ مقدار داده شده معکوس هم اند همین به ازای a و $\frac{1}{a}$ مقدار یکسان می‌دهد

$\rightarrow f(2-\sqrt{3})+f(2+\sqrt{3})=2f(2+\sqrt{3})$

$\rightarrow f(2+\sqrt{3})=\sqrt{2+\sqrt{3}+\frac{1}{2+\sqrt{3}}+2}+2=\sqrt{2+\sqrt{3}+2-\sqrt{3}+2}+2=\sqrt{6}+2$

$\rightarrow 2f(2+\sqrt{3})=4+2\sqrt{6}$

۸) $\begin{cases} 2f(x)-3f(-x)=4x^2-x & \times 2 \\ 2f(-x)-3f(x)=4x^2+x & \times 3 \end{cases} \rightarrow \begin{cases} 4f(x)-6f(-x)=8x^2-2x \\ 6f(-x)-9f(x)=12x^2+3x \end{cases}$

$-5f(x)=20x^2+x$

$\rightarrow f(x)=-4x^2-\frac{1}{5}x$

۹) $(x+2)f(x)-3xf(x+2)=4x^2-mx+3m-1$

$\xrightarrow{x=-2} 6f(0)=16+2m+3m-1$

$\xrightarrow{x=0} 2f(0)=3m-1 \xrightarrow{\times 3} 6f(0)=9m-3$

$\Rightarrow 5m+15=9m-3 \rightarrow 4m=18 \rightarrow m=\frac{18}{4}=4.5$

$\rightarrow 6f(0)=22.5+15=37.5 \rightarrow f(0)=\frac{37.5}{6}=6.25$

۱۰) $\xrightarrow{x=-1} f(-1)+f(-1)=\frac{3+12^{\cdot 12(-1)}+3}{-1}$

$\rightarrow 2f(-1)=-18 \rightarrow f(-1)=-9$