

الف) $\frac{x-1}{x} - \frac{x}{x-1} \geq 0 \Rightarrow \frac{x^2 - 2x + 1 - x^2}{x^2 - x} \geq 0 \Rightarrow \frac{-2x + 1}{x(x-1)} \geq 0$
 $\frac{10}{+} \frac{1}{-} \frac{1}{+} \frac{1}{-}$
 $D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $\frac{3}{x-1} + \frac{1}{x+2} \neq 0 \Rightarrow \frac{3x+6+x-1}{x^2+x-2} \neq 0 \Rightarrow \frac{4x+5}{(x-1)(x+2)} \neq 0$
 $x \neq -1$ / $x \neq 0$
 $D_f = \mathbb{R} - \{-2, -\frac{5}{4}, -1, 0, 1\}$

الف) $(\frac{1}{x})^x - (\frac{1}{x})^{-x} \geq 0 \Rightarrow \frac{-x}{+} \frac{1}{-} \frac{1}{+} \frac{1}{-} \Rightarrow D_f = (-\infty, -2] \cup [\frac{5}{4}, +\infty)$

ب) $\sqrt{x-1} + \sqrt{y+1} = 3$
 $x \geq 1$ / $y \geq -1$
 $\Rightarrow D_f = [1, 12]$

$x^2 - x - 2 > 0 \Rightarrow \frac{-1}{+} \frac{2}{-} \Rightarrow \frac{x < -1}{x > 2}$ / $x^2 - 1 \geq 0 \Rightarrow \frac{x \geq 1}{x \leq -1}$
 $\Rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$

$3 + ax - x^2 \geq 0 \Rightarrow 3 - 2a - 1 = -1 - 2a = 0 \Rightarrow a = -\frac{1}{2}$
 $3 + ab - b^2 = 0 \Rightarrow -b^2 - \frac{b}{2} + 3 = 0$
 $\Rightarrow a+b = -\frac{1}{2} + \frac{3}{2} = 1$

$f(x) - x \geq 0 \Rightarrow \left. \begin{array}{l} x \geq 1 \Rightarrow 3x - x - x = x^2 - x \geq 0 \Rightarrow x \geq 1 \\ x < 1 \Rightarrow 3x + 3 - x = x + 3 \geq 0 \Rightarrow x \geq -3 \end{array} \right\} D_f = [-3, +\infty)$

$x(a+b) = 3a - 4 + a \Rightarrow 14a + 14 = 4a - 4 \Rightarrow 10a = -18 \Rightarrow a = -1.8$

$f(x) = \sqrt{x + \frac{1}{\sqrt{x}}} + 2$
 $x - \sqrt{x} = \frac{1}{x + \sqrt{x}} \Rightarrow 1 = 4 - 3 \Rightarrow 1 = 1$

$\Rightarrow f(x - \sqrt{x}) + f(x + \sqrt{x}) = \sqrt{x - \sqrt{x}} + \frac{1}{\sqrt{x - \sqrt{x}}} + 2 + \sqrt{x + \sqrt{x}} + \frac{1}{\sqrt{x + \sqrt{x}}} + 2$
 $= 2\sqrt{x - \sqrt{x}} + \frac{2}{\sqrt{x - \sqrt{x}}} + 4 = 2(\sqrt{x - \sqrt{x}} + \frac{1}{\sqrt{x - \sqrt{x}}}) + 4$
 $\sqrt{x - \sqrt{x}} + \frac{1}{\sqrt{x - \sqrt{x}}} = \sqrt{x}$
 $\Rightarrow 2(\sqrt{x} + 2) = 2\sqrt{x} + 4$

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$$\left. \begin{aligned} & \left(\begin{aligned} & 4(f(x) - 3f(-x)) = 5x^2 - x \\ & 3(4f(-x) - 3f(x)) = 5x^2 + x \end{aligned} \right) \Rightarrow \left\{ \begin{aligned} & 4f(x) - 12f(-x) = 5x^2 - x \\ & 12f(-x) - 9f(x) = 5x^2 + x \end{aligned} \right. \Rightarrow \begin{cases} -5f(x) = 20x^2 + x \\ \Rightarrow f(x) = \frac{4x^2 + x}{-5} \end{cases}$$

$$\begin{aligned} x = -2 \Rightarrow 4f(0) &= 16 + 2m + 3m - 1 \Rightarrow f(0) = \frac{5m + 15}{4} \\ x = 0 \Rightarrow 4f(0) &= 3m - 1 \Rightarrow f(0) = \frac{3m - 1}{4} \end{aligned} \Rightarrow \begin{cases} 9m - 3 = 5m + 15 \Rightarrow 4m = 18 \\ m = \frac{18}{4} = \frac{9}{2} \end{cases}$$

$$\Rightarrow f(0) = \frac{13 \cdot \frac{9}{2} - 1}{4} = \frac{5, 25}{4}$$

$$\textcircled{1} \xrightarrow{x=-1} 4f(-1) = \frac{3+12+3}{-1} = -18 \Rightarrow f(-1) = -\frac{9}{2}$$

یا می توان از راه دیگر رفت که از داده سوال نیز استفاده شود.

$$\textcircled{2} f(x) + f\left(\frac{1}{x}\right) = ax + b + \frac{a}{x} + b = \frac{ax^2 + 2bx + a}{x} = \frac{3x^2 + 12x + 3}{x}$$

$$\Rightarrow a = 3, b = 6 \Rightarrow f(x) = ax + b \Rightarrow f(x) = 3x - 6 \xrightarrow{x=-1} f(-1) = -9$$

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