

الف) $F(x) = \sqrt{\frac{x-1}{x}} - \frac{x}{x-1} \geq 0 \Rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \Rightarrow \frac{1-2x}{x(x-1)} \geq 0 \Rightarrow x = \frac{1}{2} \quad x=1 \quad x=0$

$\frac{x}{x} + \frac{1}{x} - \frac{1}{x} + \frac{1}{x} \Rightarrow D_f = (-\infty, 0) \cup [\frac{1}{2}, 1) \checkmark$

ب) $\frac{1}{x+1} - \frac{2}{x} \Rightarrow \frac{x}{x-1} + \frac{1}{x+2} \neq 0 \Rightarrow \frac{x}{x-1} \neq -\frac{1}{x+2} \Rightarrow -x+1 = x^2+x+2 \Rightarrow (x-1)$
 $\frac{x}{x-1} + \frac{1}{x+2} \Rightarrow x = -\frac{3}{5} \Rightarrow D_f = \mathbb{R} - \{-\frac{3}{5}, -1, -2, 0, 1\} \checkmark$

الف) $\sqrt{(\frac{1}{x})^4 - 9} (3^x - 4^x) \Rightarrow (\frac{1}{x})^4 - 9 = 0 \Rightarrow x = \frac{1}{3} \quad 3^x - 4^x = 0 \Rightarrow x = \frac{1}{2}$

$\frac{x}{x} + \frac{1}{x} - \frac{1}{x} + \frac{1}{x} \Rightarrow D_f = (-\infty, -\frac{1}{3}] \cup [\frac{1}{2}, +\infty) \checkmark$

ب) $\sqrt{x-1} + \sqrt{y+1} = 3 \Rightarrow \sqrt{y+1} = 3 - \sqrt{x-1} \Rightarrow 3 - \sqrt{x-1} \geq 0 \Rightarrow 3 \geq \sqrt{x-1}$
 $\Rightarrow 1 \leq x \leq 10 \Rightarrow D_f = [1, 10] \checkmark$

$f(x) = \log(x^2 - x - 2) \Rightarrow x^2 - x - 2 \geq 0 \Rightarrow (x-2)(x+1) \geq 0 \Rightarrow \frac{x}{x} + \frac{1}{x} - \frac{1}{x} + \frac{1}{x} \Rightarrow (-\infty, -1) \cup (2, +\infty)$

$\sqrt{x^2-1} + 1 \Rightarrow \sqrt{x^2-1} + 1 \neq 0 \Rightarrow \sqrt{x^2-1} \neq -1 \Rightarrow x^2-1 \neq 1 \Rightarrow x^2 \neq 2 \Rightarrow x \neq \pm\sqrt{2}$
 $\Rightarrow D_f = (-\infty, -1) \cup (2, +\infty) - \{\pm\sqrt{2}\} \text{ or } (-\infty, -1) \cup (2, +\infty) - \{-\sqrt{2}\}$

$\sqrt{x^2 + ax - x^2} \Rightarrow -x^2 + ax + 3 = 0 \Rightarrow x = -3 \Rightarrow -(-3)^2 + a(-3) + 3 = 0$
 $-9 - 3a + 3 = 0 \Rightarrow -1 - 3a = 0 \Rightarrow a = -\frac{1}{3} \Rightarrow -x^2 - \frac{1}{3}x + 3 = 0 \Rightarrow (x)^2 + \frac{1}{3}x - 3 = 0$
 $\Rightarrow (x+3)(x-1/3) = 0 \Rightarrow x = -3 \quad x = 1/3 \Rightarrow D_f = [-3, 1/3] \Rightarrow a+b = -2 + 1/3$
 $\Rightarrow a+b = -5/3 \quad a+b = -\frac{1}{3} + \frac{1}{3} = 0$

$F(x) = \begin{cases} 3x-2 & x \geq 1 \\ 2x+3 & x < 1 \end{cases} \Rightarrow g(x) = \sqrt{f(x)} - x \Rightarrow f(x) - x \geq 0$

$3x-2-x \geq 0 \Rightarrow 2x-2 \geq 0 \Rightarrow 2x \geq 2 \Rightarrow x \geq 1 \Rightarrow I$
 $2x+3-x \geq 0 \Rightarrow x+3 \geq 0 \Rightarrow x \geq -3 \Rightarrow -3 \leq x < 1 \Rightarrow II$

$D_f = I \cup II \Rightarrow [-3, 1) \cup [1, +\infty) \text{ or } [-3, +\infty) \checkmark$

$$f(x) = \begin{cases} (x+1)(x+1) & x \geq 1 \\ x^2 + 1 & x < 1 \end{cases}$$

$$f(x) = \begin{cases} (x+1)(x+1) & x \geq 1 \\ x^2 + 1 & x < 1 \end{cases} \Rightarrow f(x) = f(-x) + \alpha \Rightarrow x=0 \Rightarrow f(f(0)) = f(1+0)$$

$$\Rightarrow (1+1)^2 = 9 \quad x = -1 \Rightarrow x^2 + 1 = 1 \Rightarrow 1+1 = 1 \Rightarrow 1+1 = 1$$

$$\Rightarrow 1+1 = 1 \Rightarrow \alpha = -1/1 \quad \checkmark$$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 1 \Rightarrow f(\sqrt{x}) + f(\frac{1}{\sqrt{x}}) \Rightarrow \sqrt{x} = \alpha = \sqrt{1+1} \quad \frac{1}{\alpha} + \frac{1}{\alpha} = \frac{\alpha+1}{\alpha} = \alpha+1$$

$$\sqrt{x} = b = \sqrt{1+1} \Rightarrow \alpha b = \sqrt{1+1} = 1 \quad \alpha^2 + b^2 = (1+1)(1+1) = 4$$

$$f(x_1) + f(x_2) = (\alpha + \frac{1}{\alpha} + 1) + (b + \frac{1}{b} + 1) = (\alpha + b) + (\frac{1}{\alpha} + \frac{1}{b}) + 2 \Rightarrow (\alpha + b) + (\alpha + b) + 2$$

$$\Rightarrow f(\alpha + b) + 2 \Rightarrow (\alpha + b)^2 = \alpha^2 + b^2 + 2\alpha b = 4 + 2 \cdot 1 = 6 \Rightarrow \alpha + b = \sqrt{6}$$

$$f(x) - f(-x) = \epsilon x^p - q \Rightarrow f(-x) - f(x) = \epsilon x^p - q$$

$$\Rightarrow \begin{cases} f(x) - f(-x) = \epsilon x^p - q \\ -f(x) + f(-x) = \epsilon x^p - q \end{cases} \Rightarrow (\epsilon - q)f(x) = (\epsilon x^p - q) + (\epsilon x^p - q)$$

$$\Rightarrow -2f(x) = 2\epsilon x^p - 2q \Rightarrow f(x) = -(\epsilon x^p - q) \Rightarrow f(x) = -\epsilon x^p + q \quad \checkmark$$

$$(x+1)f(x) - xf(x+1) = \epsilon x^p - mx + m - 1$$

$$\Rightarrow x=0 \Rightarrow (1)(f(0)) - (0)(f(1+1)) = m-1 \Rightarrow f(0) = \frac{m-1}{1}$$

$$x = -1 \rightarrow f(0) = \frac{m+1}{1}$$

$$\frac{m-1}{1} = \frac{m+1}{1} \rightarrow m = \frac{1}{1} \quad f(0) = \frac{1+1}{1} = 2$$

$$f(x) + f(\frac{1}{x}) = \frac{x^p - 1}{x} \Rightarrow f(x) = ax + b \quad f(\frac{1}{x}) = \frac{a}{x} + b$$

$$\Rightarrow f(x) + f(\frac{1}{x}) = ax + b + \frac{a}{x} + b \Rightarrow \frac{ax^p}{x} + \frac{a}{x} + \frac{bx}{x} \Rightarrow \frac{ax^p + bx + a}{x} = \frac{x^p - 1}{x}$$

$$a = 1 \quad b = -1 \Rightarrow f(-1) = -1 + b = -1 - 1 = -2 \quad \checkmark$$