

$$f(x) = \left(\frac{1}{x}\right)^{x-1} \Rightarrow f(x+1) = \left(\frac{1}{x+1}\right)^{x-1} - 1$$

$$y = \sqrt{x^x f(x+1)} \Rightarrow x^x f(x+1) \geq 0 \Rightarrow x^x \times (x^{1-x} - 1) \geq 0$$

$$\begin{array}{c} \cdot \quad 1 \\ - \quad + \quad - \end{array} \quad D_f = [0, 1]$$

$$f(x) = \sqrt{x + (x+2)} \Rightarrow f(-x) = \sqrt{-x + (-x+2)} \Rightarrow -x + (-x+2) \geq 0$$

$$\begin{array}{c} (-x+2) = (x-2) \\ \Rightarrow -x + (x-2) \geq 0 \end{array} \Rightarrow \begin{array}{c} 2 \\ -2x+2 \geq 0 \\ -2x \geq -2 \\ x \leq 1 \end{array} \Rightarrow (-\infty, 1]$$

$$y = \sqrt{\frac{x-1}{f(x)}} \rightarrow \frac{x-1}{f(x)} \geq 0$$

$$\begin{array}{c} -1 \quad 1 \quad 2 \quad 3 \\ - \quad + \quad - \quad + \end{array}$$

$$D_f = (-1, 1] \cup (2, 3)$$

$$y_1 = \frac{1}{9x^4 - 12x^3 - 4x^2 - 3} = \frac{1}{(9x^4 - 12x^3 + 1) - (x^3 + 4x^2 + 4)} = \frac{1}{(3x^2-1)^2 - (x+2)^2} = \frac{1}{(3x^2+x+1)(3x^2-x-3)}$$

$$\Rightarrow D_{y_1} = \mathbb{R} - \{3x^2-x-3 \text{ رضای} \} = D_{y_2} = \mathbb{R} - \{3x^2+ax+b \text{ رضای} \}$$

$$\Rightarrow 3x^2-x-3 = 3x^2+ax+b \Rightarrow \begin{cases} a = -1 \\ b = -3 \end{cases} \Rightarrow a+b = -4 \quad -(a+b) = 4 \quad \sqrt{4} = 2$$

$$f(x) = x^x, x$$

$$f\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^{\frac{1}{x}} \cdot \frac{1}{x} = \frac{1}{x^{\frac{1}{x}+1}}$$

$$\left. \begin{array}{l} f(x) - f\left(\frac{1}{x}\right) = x^x \cdot x - \frac{1}{x^{\frac{1}{x}+1}} \\ y = \sqrt{f(x) - f\left(\frac{1}{x}\right)} \Rightarrow f(x) - f\left(\frac{1}{x}\right) \geq 0 \end{array} \right\} \Rightarrow \left(x^x - \frac{1}{x^{\frac{1}{x}+1}}\right) + \left(x - \frac{1}{x}\right) \geq 0$$

$$\Rightarrow \left(x - \frac{1}{x}\right) \left(x^x + \frac{1}{x^{\frac{1}{x}+1}}\right) + \left(x - \frac{1}{x}\right) \geq 0 \Rightarrow \left(x - \frac{1}{x}\right) \left(x^x + \frac{1}{x^{\frac{1}{x}+1}} + 1\right) \geq 0$$

$$D_f = \mathbb{R} - (-2, 2)$$

$x = \frac{1}{x} \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

$x^4 + 12x^3 + 14 \Rightarrow$ رضای ندارد
همواره مثبت

$$f(x^r, x) = r x^{r-1}$$

$$\xrightarrow{x=1} f(1,1) = r(1)^{r-1} \Rightarrow f(r) = 1$$

$$\xrightarrow{x=r} f(1,r) = r(r)^{r-1} \Rightarrow f(1,r) = r$$

$$\Rightarrow f(r) + f(1,r) = 1 + r = \lambda$$

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$$f(x) = x^r - 4x^r + 12x = x^r - 4x^r + 12x - \lambda + \lambda \Rightarrow (x-r)^r + \lambda \Rightarrow f(\sqrt{r}, r) = (\sqrt{r})^r + \lambda = 10$$

$$g(x) = x^r + 9x^r + 2rx = x^r + 9x^r + 2rx - 2r + 2r = (x+r)^r - 2r \Rightarrow g(\sqrt{r}, r) = (\sqrt{r})^r - 2r = -2$$

$$\frac{y_0}{10} = -2$$

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$$f(x) = \sqrt{x \cdot \varepsilon \sqrt{x \cdot \varepsilon}} = \sqrt{x \cdot \varepsilon + \varepsilon \sqrt{x \cdot \varepsilon}} = \sqrt{(\sqrt{x \cdot \varepsilon} + \varepsilon)^2} = |\sqrt{x \cdot \varepsilon} + \varepsilon| = \sqrt{x \cdot \varepsilon} + \varepsilon$$

$$g(x) = \sqrt{x \cdot \varepsilon \sqrt{x \cdot \varepsilon}} = \sqrt{x \cdot \varepsilon - \varepsilon \sqrt{x \cdot \varepsilon}} = \sqrt{(\sqrt{x \cdot \varepsilon} - \varepsilon)^2} = |\sqrt{x \cdot \varepsilon} - \varepsilon| \Rightarrow \begin{cases} \sqrt{x \cdot \varepsilon} - \varepsilon = x \wedge \\ -\sqrt{x \cdot \varepsilon} + \varepsilon = \varepsilon \wedge x \leq \varepsilon \end{cases}$$

$$\Rightarrow f(x) + g(x) = \begin{cases} a & b \\ 0 + \varepsilon \sqrt{x \cdot \varepsilon} & x \geq \varepsilon \Rightarrow \frac{a+b}{c} = \frac{\varepsilon}{-\varepsilon} = -\frac{1}{\varepsilon} \\ \varepsilon + 0 \sqrt{x \cdot \varepsilon} & \varepsilon < x < \varepsilon \Rightarrow \frac{a+b}{c} = \frac{\varepsilon}{-\varepsilon} = -1 \end{cases}$$

$$g(x) = \left\{ (1,1), (1,0), (r,0), (0,\varepsilon) \right\} \Rightarrow Dg = \{1, r, 0, \varepsilon\}$$

$$f(x) = \frac{x+r}{x^r - \varepsilon x + r} \Rightarrow Df = \mathbb{R} - \{1, r\}$$

$$f(r) = -\varepsilon$$

$$f(0) = \frac{r}{r}$$

$$\frac{g}{f} = \left\{ (r, -\frac{1}{\varepsilon}), (0, r) \right\}$$

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$$\text{الف) } f(rx) = \left\{ \left(\frac{1}{r}, r\right), \left(\frac{r}{r}, -1\right), (r, r), \left(-\frac{1}{r}, r\right) \right\}$$

$$\text{ب) } f(x^r) = \left\{ (1, r), (-1, r), (\sqrt{r}, -1), (-\sqrt{r}, -1), (r, r), (-r, r) \right\}$$

$$\text{ج) } r g'(x) + 1 = \left\{ (-2, 19), (1, 3), (3, 9), (-1, 1) \right\}$$

$$\text{د) } \frac{rf}{g} = \left\{ (1, -\varepsilon), (r, -1) \right\}$$

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