Subject.
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تکلیف ۵ — امیر پویا رزاق

(۱) $f(x) = (\frac{1}{2})^{x-2} - 1$ ، $y = \sqrt{x^3 f(x+1)}$ ، $D_y = ?$

$x^3 \times (\frac{1}{2})^{x+1-2} - 1 = x^3 f(x+1) \geq 0 \rightarrow$ زیر رادیکال $\Rightarrow x^3 \geq 0 \Rightarrow \boxed{x \geq 0}$ ①

$(\frac{1}{2})^{x+1-2} - 1 \geq 0 \Rightarrow (\frac{1}{2})^{x-1} - 1 \geq 0 \Rightarrow (\frac{1}{2})^{x-1} \geq 1 \Rightarrow x-1 \leq 0 \rightarrow \boxed{x \leq 1}$ ②

$(\frac{1}{2})^0 = 1$

$D_f = ① \cap ② \Rightarrow D_f = [0,1]$

(۲) $f(x) = \sqrt{x+1\,|x+2|}$ ، $D_{f(-x)} = ?$

$f_{(-x)} = \begin{cases} 2-x \geq x \Rightarrow 2 \geq 2x \Rightarrow 1 \geq x \;;\; x \leq 2 \;\checkmark \\ x-2 \geq x \Rightarrow -2 \geq 0 \;;\; x > 2 \;\times \text{ تعریف نشده} \end{cases}$

$\Rightarrow \boxed{D_f = (-\infty, 1]}$

(۳) $y = \sqrt{\frac{x-1}{f(x)}}$ ، $D_y = ?$

شرط ۱ $\rightarrow \frac{x-1}{f(x)} \geq 0$

شرط ۲ $\rightarrow f(x) \neq 0$

ریشه‌های تابع $\Rightarrow x = -5, -4, 2, 3$

	-5	-4	2	3	
	+	−	−	+	−

$x \geq 1 \Rightarrow f(x) > 0 \Rightarrow (2,3) \cup (3,+\infty)$ ①

$x < 1 \Rightarrow f x < 0 \Rightarrow (-5,-4) \cup (-4,1)$

$\mathbb{R} - \{-5, -4, 2, 3\}$ ②

① ∩ ② =

$\boxed{D_f = (-4,1] \cup (2,3)}$

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④ $y_1 = \frac{1}{9x^2 - 7x^2 - 6x - 3} \Rightarrow y_1 = \frac{1}{(3x^2+x+1)(3x^2-x-3)}$

$3x^2 + x + 1 > 0 \Rightarrow \Delta = 1 - 12 < 0$

$\Rightarrow 3x^2 - x - 3 = 3x^2 + ax + b$

$3x^2 - x - 3 > 0 \Rightarrow \Delta = 1 + 36 > 0$

$\Rightarrow a = -1,\ b = -3 \Rightarrow \sqrt{-(a+b)} = \sqrt{-(-1-3)} \Rightarrow \sqrt{-(-4)} = \boxed{\sqrt{4} = 2}$

⑤ $f(x) = x^3 + x$, $y = \sqrt{f(x) - f(\frac{4}{x})}$, $D_y = ?$

$f(x) - f(\frac{4}{x}) = (x^3 + x) - (\frac{64}{x^3} + \frac{4}{x}) = \frac{x^6 + x^4 - 4x^2 - 64}{x^3}$ ← مخرج مشترک

صورت $= x^6 + x^4 - 4x^2 - 64 = (x-2)(x+2)(x^4 + 5x^2 + 16)$

$\Rightarrow y = \sqrt{\frac{(x-2)(x+2)(x^4+5x^2+16)}{x^3}}$ ① $x \neq 0$ شرط ۱

$\Rightarrow$ زیر رادیکال شرط ۲ ≥ 0

$\Rightarrow x^4 + 5x^2 + 16 \rightarrow \Delta < 0,\ a > 0 \Rightarrow$ ریشه = $\boxed{2, -2}$

	-2		2	
	$+$	$-$	$+$	

② $\boxed{(-2, 2)}$

$\Rightarrow \boxed{D_y = [-\infty, -2) \cup [2, +\infty)}$

⑥ $f(x^3 + x) = 2x^2 - 1$, $f(2) + f(10) = ?$

$f(x^3 + x) = f(10) \Rightarrow x^3 + x = 10 \Rightarrow x^3 + x - 10 = 0 \Rightarrow x = 2 \Rightarrow$

$f(10) = 2(4) - 1 = \boxed{7}$, $f(x^3 + x) = f(2) \Rightarrow x^3 + x - 2 = 0 \Rightarrow \boxed{x = 1} \Rightarrow$

$f(2) = 2(1) - 1 \rightarrow ① \Rightarrow \boxed{f(10) + f(2) = 7 + 1 = 8}$

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⑦ $f(x) = x^3 - 4x^2 + 12x$

$g(x) = x^3 - 9x^2 + 27x$, $\frac{g(\sqrt[3]{7}-2)}{f(\sqrt[3]{2}+2)} = ?$

$x_1 = \sqrt[3]{7} - 2$, $x_2 = \sqrt[3]{4} + 2$, $g(\sqrt[3]{7}-2) \Rightarrow x_1 + 2 = \sqrt[3]{7}$

$\Rightarrow g(x_1) = (x_1+2)^3 - 27 \Rightarrow 7 - 27 = -20$

$f(\sqrt[3]{4}+2) \rightarrow x_2 - 2 = \sqrt[3]{4} \Rightarrow f(x_2) = (x_2-2)^3 + 8 \Rightarrow 4 + 8 = 12$

$\frac{g(\sqrt[3]{7}-2)}{f(\sqrt[3]{4}+2)} = \frac{-20}{12} = \frac{-5}{3} \Rightarrow \boxed{\frac{-5}{3}}$

⑧ $f(x) = \sqrt{x+4\sqrt{x-4}}$

$g(x) = \sqrt{x-4\sqrt{x-4}}$ $\xrightarrow{x \geq 4}$ $f(x) + g(x) = a + b\sqrt{x+c}$

$\frac{a+b}{c} = ?$ $f(x) + g(x) = \sqrt{x} + 2\sqrt{x-4} + \sqrt{x}\sqrt{x-4} - 4$

$\Rightarrow a = 1$, $b = 4$, $c = -4 \Rightarrow \frac{a+b}{c} = \frac{1+4}{-4} = \boxed{\frac{-5}{4}}$

⑨ $f(x) = \frac{x+2}{x^2-4x+2}$, $g(x) \{(2,1),(1,0),(2,5),(0,4)\}$

$\frac{g}{f} = ?$ $D_f = x \neq 1, 2 = \mathbb{R} - \{1, 2\}$

$D_g = 0, 2$ حذف شدن $\Big\}$ $\xrightarrow{x=2}$ $g(2) = 1$, $f(2) = \frac{2+2}{4-4+2} = \frac{4}{-1} = \boxed{-4}$

$\frac{g}{f}(0) = \frac{4}{\frac{2}{2}} = 4$ $\xrightarrow{x=0}$ $g(0) = 4$, $f(0) = \frac{0+2}{0-0+2} = \frac{2}{2}$

$\Rightarrow \boxed{\frac{g}{f} = \{(2, -\frac{1}{4}), (0, 4)\}}$

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10) $f(x) = \{(1,2), (3,-1), (4,2), (6,9)\}$

$g(x) = \{(-2,3), (1,-1), (3,2), (-1,0)\}$

ب) $f(x) = \{(1,2), (-1,2), (\sqrt{2},-1), (-\sqrt{2},-1), (2,2), (-2,2)\}$

الف) $f(2x) = \{(\frac{1}{2},2), (\frac{3}{2},-1), (2,2), (-\frac{1}{2},6)\}$

ج) $2g^2(x)+1 = \{(-2,19), (1,3), (3,9), (1,1)\}$

د) $\frac{2f}{g} \Rightarrow 2f = \{(1,4), (3,-2), (4,4), (-1,12)\} = \{(1,-4), (3,-1)\}$

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