

$$y = \sqrt{x^3 \left(\frac{1}{x}\right)^{x+1} - 1} \Rightarrow y = \sqrt{x^3 \left(\frac{1}{x}\right)^{x-1} - 1} \Rightarrow x^3 \left(\frac{1}{x}\right)^{x-1} - 1 \geq 0$$

$$x^3 \left(\frac{1}{x}\right)^{x-1} \geq 1 \Rightarrow x \geq 1 \Rightarrow x^3 \left(\frac{1}{x}\right)^{x-1} \geq 1 \Rightarrow Df = [1, +\infty)$$

اما x^3 به ازای اعداد منفی، مثبت است
چون $\left(\frac{1}{x}\right)^{x-1}$ همواره مثبت است
چون x^3 در بازه $(0, +\infty)$ همواره مثبت است
از یک است

$$f(-x) = -x + |2-x| \quad -x + |2-x| \geq 0$$

$$x \geq 2 \rightarrow -x + x - 2 = -2 < 0 \quad \text{غیر}$$

$$x < 2 \rightarrow -x + 2 - x = -2x + 2 \geq 0 \quad 2(-x+1) \geq 0 \quad \frac{2}{-x+1} \geq 1 \quad (x \leq 1)$$

$$\Rightarrow (-\infty, 1] \cap (-\infty, 2) = (-\infty, 1] \Rightarrow Df = (-\infty, 1]$$

$$y = \sqrt{\frac{x-1}{f(x)}} \Rightarrow \frac{x-1}{f(x)} \geq 0$$

$$Df = (-\infty, 1] \cup (2, 3)$$

$$\text{① حالت} \Rightarrow x-1 \text{ و } f(x) \text{ هر دو مثبت} \rightarrow (2, 3) \cap [(2, 3) \cup [-5, -1]] = (2, 3)$$

$$\text{② حالت} \Rightarrow x-1 = 0, f(x) \neq 0 \rightarrow x-1=0 \Rightarrow x=1, f(x) \neq 0 \Rightarrow x \neq \{-5, 2, 3\}$$

$$\text{③ حالت} \Rightarrow x-1, f(x) \text{ هر دو منفی} \rightarrow (-5, 1) \cap (-5, 2) = (-5, 1)$$

$$9x^2 - 4x^2 + 1 - x^2 - x^2 - x^2 = 4x^2 - 4x^2 + 1 - x^2 - x^2 - x^2 = 1 - 3x^2$$

$$(3x^2 - 1)^2 - (x+2)^2 = (3x^2 - 1 + x + 2)(3x^2 - 1 - x - 2) = (3x^2 + x + 1)(3x^2 - x - 3) \rightarrow \Delta < 0$$

$$3x^2 - x - 3 = 3x^2 + ax + b \Rightarrow a = -1, b = -3 \quad \sqrt{-(-1-3)} = \sqrt{4} = 2$$

$$y = \sqrt{x^3 + x - \frac{4x}{x^3} - \frac{4}{x}} \Rightarrow \frac{x^4 + x^2 - 4x^2 - 4x^2}{x^3} = \frac{x^4 - 6x^2 + 4x^2}{x^3} = \frac{x^4 - 2x^2}{x^3}$$

$$(x^2 - 2)(x^2 + 2) + x^2(x^2 - 2) = (x-2)(x^2 + 2x + 4)(x+2)(x^2 + 2x + 4) + (x-2)(x+2)(x^2 + 2x + 4)$$

$$(x-2)(x+2)((x^2 + 2x + 4)(x^2 + 2x + 4) + x^2) = (x-2)(x+2)(x^4 + 4x^3 + 14x^2 + 14x + 4)$$

$$x = \pm 2 \quad Df = \mathbb{R} - (-2, 2) \quad \Delta < 0, a > 0 \quad \text{همواره مثبت}$$

$$x^3 \neq 0 \Rightarrow x \neq 0$$

$$\frac{x}{y} \mid \begin{array}{c} -2 \\ +1 \\ -1 \\ + \end{array}$$

من اعداد ۲ و ۱۰ را به فرم $x^3 + x$ نوشتیم و مقدار x را در تابع جایگذاری کردیم:

$$\left. \begin{aligned} x^3 + x = 2 \quad (x=1) &\rightarrow 1^3 + 1 = 2 & f(1) = 1 \\ x^3 + x = 10 \quad (x=2) &\rightarrow 2^3 + 2 = 10 & f(2) = 2 \end{aligned} \right\} f(1) + f(10) = 1$$

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$f(x) = x^3 - 4x^2 + 12x - 8 + 8 \Rightarrow (x-2)^3 + 8 = f(x)$
 اتحاد مکعب دو جمله

$g(x) = x^3 + 9x^2 + 27x + 27 - 27 \Rightarrow (x+3)^3 - 27 = g(x)$
 اتحاد مکعب دو جمله

$$\frac{g(\sqrt[3]{27-27})}{f(\sqrt[3]{27+27})} = \frac{(\sqrt[3]{27-27})^3 - 27}{(\sqrt[3]{27+27})^3 + 8} = \frac{27-27}{27+8} = \frac{-27}{35} = -\frac{27}{35}$$

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$$(f(x) + g(x))^2 = x - 4\sqrt{x-4} + x + 4\sqrt{x-4} + 2\sqrt{\frac{x^2 - 14x + 48}{x^2 - 14x + 48} = (x-4)^2}$$

$$2x + 2\sqrt{(x-4)^2} = 2x + 2|x-4|$$

$4 \leq x < 8 \rightarrow 2x + 2(8-x) = 2x + 16 - 2x = 16 \rightarrow$ مشابه به مباحثات صورت سوال نیست

$x \geq 8 \rightarrow 2x + 2x - 16 = 4x - 16 \rightarrow \sqrt{4x-16} = \sqrt{4(x-4)} = 2\sqrt{x-4} \Rightarrow \frac{2+0}{-2} = -\frac{1}{2}$

$\begin{cases} a=0 \\ b=2 \\ c=-4 \end{cases}$

جواب $\uparrow$
 $-\frac{1}{2}$

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$f(x) = \frac{x+7}{(x-3)(x-1)}$ $Df = \mathbb{R} - \{3, 1\}$ $Dg = \{0, 1, 3, 0\}$ $\Rightarrow D_{مشترک} = \{2, 0\}$

$$\frac{g}{f} = \left\{ (0, 4), (2, -\frac{1}{4}) \right\}$$

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الف) $f(2x) = \left\{ (\frac{1}{2}, 2), (\frac{3}{2}, -1), (2, 2), (-\frac{1}{2}, 4) \right\}$

ب) $f(x^2) = \{(1, 2), (-1, 2), (-\sqrt{3}, -1), (\sqrt{3}, -1), (2, 2), (-2, 2)\}$

ج) $2g'(x) + 1 = \{(-2, 19), (1, 3), (3, 9), (-1, 1)\}$

د) $\frac{2f}{g} = \{(2, -4), (3, -1), (-\frac{1}{2}, \frac{14}{3})\} = \{(2, -4), (3, -1)\}$

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