

$$y = \sqrt{x^3 f(x+1)} \Rightarrow f(x+1) = \left(\frac{1}{x}\right)^{(x+1)-2} \Rightarrow \left(\frac{1}{x}\right)^{x-1}$$

$$\Rightarrow x^3 f(x+1) \Rightarrow x^3 \left(\left(\frac{1}{x}\right)^{x-1} - 1\right) \Rightarrow x^3 \left(\left(\frac{1}{x}\right)^{x-1} - 1\right) \geq 0$$

$$\Rightarrow x^3 = 0 \Rightarrow x = 0 \quad \text{و} \quad \left(\frac{1}{x}\right)^{x-1} - 1 = 0 \Rightarrow \left(\frac{1}{x}\right)^{x-1} = 1 \Rightarrow x = 1$$

گ $\frac{x}{x-1} + \frac{1}{x-1} \Rightarrow D_f = [0, 1]$

$$f(x) = \sqrt{x+|x+2|} \Rightarrow f(-x) = \sqrt{-x+|2-x|} \Rightarrow -x+|2-x| \geq 0$$

$$\Rightarrow |2-x| \Rightarrow x \leq 2 \Rightarrow |2-x| = 2-x \Rightarrow -x+(2-x) = 2-2x$$

$$\Rightarrow 2-2x \geq 0 \Rightarrow x \leq 1 \quad \text{و} \quad |2-x| \Rightarrow x > 2 \Rightarrow |2-x| = x-2 \Rightarrow -x+(x-2) = -2 \Rightarrow -2 = -2 \checkmark$$

د $f(-x)$ در $D_f = (-\infty, 1]$

$$g = \sqrt{\frac{x-1}{f(x)}} \Rightarrow x-1=0 \Rightarrow x=1 \quad \text{و} \quad f(x)=0 \Rightarrow x=2, 3, \dots$$

گ $\frac{x}{x-1}$ $\frac{x-1}{f(x)}$ $\Rightarrow D_f = [-\varepsilon, 1] \cup (2, 3)$

$$y_1 = \frac{1}{x^3 - \sqrt{x^3 - \varepsilon x - 3}} \quad \text{و} \quad y_2 = \frac{1}{x^3 + \alpha x + b} \Rightarrow D_{f_{y_1}} = D_{f_{y_2}}$$

$$\Rightarrow y_1 = \frac{1}{(x^3 - \alpha x - 3)(x^3 + \alpha x + b)} \Rightarrow x^3 + \alpha x + 1 \Rightarrow \Delta < 0 \Rightarrow$$

د $x^3 - \alpha x - 3 = x^3 + \alpha x + b \Rightarrow \alpha = -1 \quad b = -3 \Rightarrow \sqrt{-(\alpha+b)} = \sqrt{-(-1-3)} \Rightarrow \sqrt{4} = \pm 2$

$$f(x) = x^3 + x \Rightarrow y = \sqrt{f(x) - f\left(\frac{x}{\varepsilon}\right)} \Rightarrow \left(x^3 + x\right) - \left(\left(\frac{x}{\varepsilon}\right)^3 + \frac{x}{\varepsilon}\right)$$

$$\Rightarrow x^3 + x - \frac{x^3}{\varepsilon^3} - \frac{x}{\varepsilon} \Rightarrow x^3 \left(1 - \frac{1}{\varepsilon^3}\right) + x \left(1 - \frac{1}{\varepsilon}\right) \Rightarrow \frac{x^3}{\varepsilon^3} x^3 + \frac{x}{\varepsilon} x$$

$$\Rightarrow \frac{x^3}{\varepsilon^3} x^3 + \frac{x}{\varepsilon} x \Rightarrow x \left(\frac{x^3}{\varepsilon^3} x^2 + \frac{x}{\varepsilon}\right) \Rightarrow x \left(\frac{x^3 x^2 + \varepsilon x}{\varepsilon^3}\right) \Rightarrow x \left(\frac{x^5 + \varepsilon x}{\varepsilon^3}\right)$$

د $y = \sqrt{\frac{x(x^4 + \varepsilon)}{\varepsilon^3}} \Rightarrow \sqrt{\frac{x(x^4 + \varepsilon)}{\varepsilon^3}} \Rightarrow \frac{x^3 x^4 + \varepsilon x}{\varepsilon^3} \quad \text{و} \quad x=0 \Rightarrow D_f = [0, \infty)$

$$f(x+y) = 2xy - 1 \Rightarrow f(10) + f(1) = ?$$

$$\Rightarrow x+y = y \Rightarrow f(y) = 2xy - 1 \Rightarrow xy = 10 \Rightarrow x+y - 10 = 0 \Rightarrow x = 1 \Rightarrow 2(1)y - 1 = 10$$

$$f(1) \Rightarrow xy = 1 \Rightarrow xy - 1 = 0 \Rightarrow x = 1 \Rightarrow 2(1)y - 1 = 10 \Rightarrow y = 1$$

$$\Rightarrow f(10) + f(1) = 10 + 1 = 11$$

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$$f(x) = x^3 - 9x^2 + 17x \Rightarrow (x-2)^3 = x^3 - 9x^2 + 17x - 8 \Rightarrow f(x) = (x-2)^3 + 8$$

$$g(x) = x^3 + 9x^2 + 17x \Rightarrow (x+2)^3 = x^3 + 9x^2 + 17x + 8 \Rightarrow g(x) = (x+2)^3 - 8$$

$$x = \sqrt[3]{10} - 2 \Rightarrow (\sqrt[3]{10} + 2 - 2)^3 - 8 = 10 - 8 = 2$$

$$f(\sqrt[3]{10} + 2) = (\sqrt[3]{10} + 2 - 2)^3 + 8 = 10 + 8 = 18$$

$$(\sqrt[3]{10})^3 + 8 \Rightarrow 10 + 8 = 18 \Rightarrow \frac{g(\sqrt[3]{10} - 2)}{f(\sqrt[3]{10} + 2)} = \frac{-2}{18} = -\frac{1}{9}$$

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$$f(x) = \sqrt{x + \epsilon \sqrt{x - \epsilon}} \text{ , } g(x) = \sqrt{x - \epsilon \sqrt{x - \epsilon}} \text{ , } x > \epsilon \Rightarrow f(x) + g(x) = a + b\sqrt{x + c}$$

$$(f+g)^2 \Rightarrow f^2 + g^2 + 2fg \Rightarrow f^2 + g^2 = (x + \epsilon \sqrt{x - \epsilon}) + (x - \epsilon \sqrt{x - \epsilon}) = 2x$$

$$fg = \sqrt{(x + \epsilon \sqrt{x - \epsilon})(x - \epsilon \sqrt{x - \epsilon})} = \sqrt{x^2 - \epsilon^2(x - \epsilon)} = \sqrt{x^2 - \epsilon x + \epsilon^2} = x - \epsilon \Rightarrow (f+g)^2 = 2x + 2(x - \epsilon) = 4x - 2\epsilon$$

$$f+g = \sqrt{4x - 2\epsilon} \Rightarrow 2\sqrt{x - \epsilon} \Rightarrow \sqrt{x - \epsilon} = a + b\sqrt{x + c}$$

$$a = 0 \quad b = 2 \quad c = -\epsilon \quad \frac{a+b}{c} = \frac{2}{-\epsilon} = -\frac{2}{\epsilon}$$

$$f(x) = \frac{x+2}{x^2 - 5x + 3} \quad g(x) = \{(1, 1), (1, 0), (2, 0), (0, 1)\} \quad \frac{g}{f} = ?$$

$$x^2 - 5x + 3 = 0 \Rightarrow (x-3)(x-1) \Rightarrow x=1 \text{ or } x=3$$

$$x+2=0 \Rightarrow x=-2 \Rightarrow \frac{g}{f} \Rightarrow f = -2 = 0$$

$$\frac{g}{f} = \{(1, \frac{1}{-2}), (0, \frac{1}{3})\} \Rightarrow \frac{g}{f} = \{(1, -\frac{1}{2}), (0, \frac{1}{3})\}$$

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$$1) f(x) = \{(1, 2), (\frac{1}{2}, -1), (2, 2), (-\frac{1}{2}, 2)\}$$

$$2) f(x^2) = \{(1, 2), (\sqrt{3}, -1), (2, 2), (-2, 2), (-1, 2)\}$$

$$3) f(g(x)+1) = \{(-2, 1), (1, 3), (3, 0), (-1, 1)\}$$

$$4) \frac{xf}{y} = \frac{\{(1, 4), (2, -2), (3, 3), (-1, 1)\}}{\{(1, 0), (1, -1), (2, 2), (-1, 0)\}} \Rightarrow \{(1, -4), (2, -1), (3, 1), (-1, 1)\}$$

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