

(4)

(1)

الف) $y = n^3 - 3n^2 + 3n$

$y = (n-1)^3 + 1 \rightarrow y-1 = (n-1)^3$

$\sqrt[3]{y-1} + 1 = n$

$D_y = \mathbb{R} \Rightarrow R_f = \mathbb{R}$

ب) $y = \frac{1}{n^2 - 2n}$

$yn^2 - 2yn - 1 = 0$

$\Delta \geq 0 \rightarrow 4^2 + 4 \geq 0 \rightarrow 4y(y+1) \geq 0$

$\begin{array}{c} -1 \quad 0 \\ + \quad - \quad + \end{array}$

$D_y = (-\infty, -1] \cup [0, +\infty)$

$R_f = (-\infty, -1] \cup [0, +\infty)$

الف) $y = n^2 - 6n + 1$

$\min \left| -\frac{b}{2a} = \frac{3}{1} = 3 \right| \rightarrow R_f = [9, +\infty)$

ب) $y = -n^2 + 9n + 3$

$\max \left| -\frac{b}{2a} = \frac{9}{2} = 4.5 \right| \rightarrow R_f = (-\infty, 12]$

2) $y = \sqrt{n^2 - 6n - 3}$

$\min \left| -\frac{b}{2a} = \frac{3}{1} = 3 \right|$

$R = [-4, +\infty)$

$\sqrt{[-4, +\infty)} \Rightarrow R_f = [0, +\infty)$

3) $y = \sqrt{9n - n^2}$

$\max \left| -\frac{b}{2a} = \frac{9}{2} = 4.5 \right|$

$R = (-\infty, 9]$

$\sqrt{(-\infty, 9]} \rightarrow R_f = [0, 3]$

(2)

الف) $y = n^3 - \omega n^2 + 2n + 1$

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مشتق مرتبه فرد $\rightarrow R_f = \mathbb{R}$

ب) $y = n^4 - 4n^3 + 3n + 2$

مشتقات مرتبه فرد $\rightarrow R_f = \mathbb{R}$

ج) $y = \sqrt{n^4 - 4n^3 + \omega n + 1}$

$n^4 - 4n^3 + \omega n + 1 \rightarrow n^4$ مشتقات مرتبه فرد $\Rightarrow R = \mathbb{R}$

$y = \sqrt{(-\infty, +\infty)} \rightarrow R_f = [0, +\infty)$

د) $y = (n^5 - 5n^4 + 3n + 1)^2$

$n^5 - 5n^4 + 3n + 1 \rightarrow n^5$ مشتقات مرتبه فرد

$R \supset \mathbb{R} \Rightarrow (n^5 - 5n^4 + 3n + 1)^2 \rightarrow (-\infty, +\infty)$

الف) $y = \frac{2n+1}{n-2}$

$R_f = [0, +\infty)$

$R_f = \mathbb{R} - \left\{\frac{a}{c}\right\} \Rightarrow R_f = \mathbb{R} - \{2\}$

(۴)

ب) $y = \frac{2n+1}{n+1} \rightarrow R_f = \mathbb{R} - \left\{\frac{a}{c}\right\} \rightarrow R_f = \mathbb{R} - \{2\}$

الف) $y = \sqrt{\frac{4n+5}{n+1}}$

$\sqrt{\mathbb{R} - \{2\}} \rightarrow R_f = [0, +\infty) - \{\sqrt{2}\}$

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ب) $y = \sqrt{\frac{2n+1}{2-n}}$ $\Rightarrow R_f = [0, +\infty)$

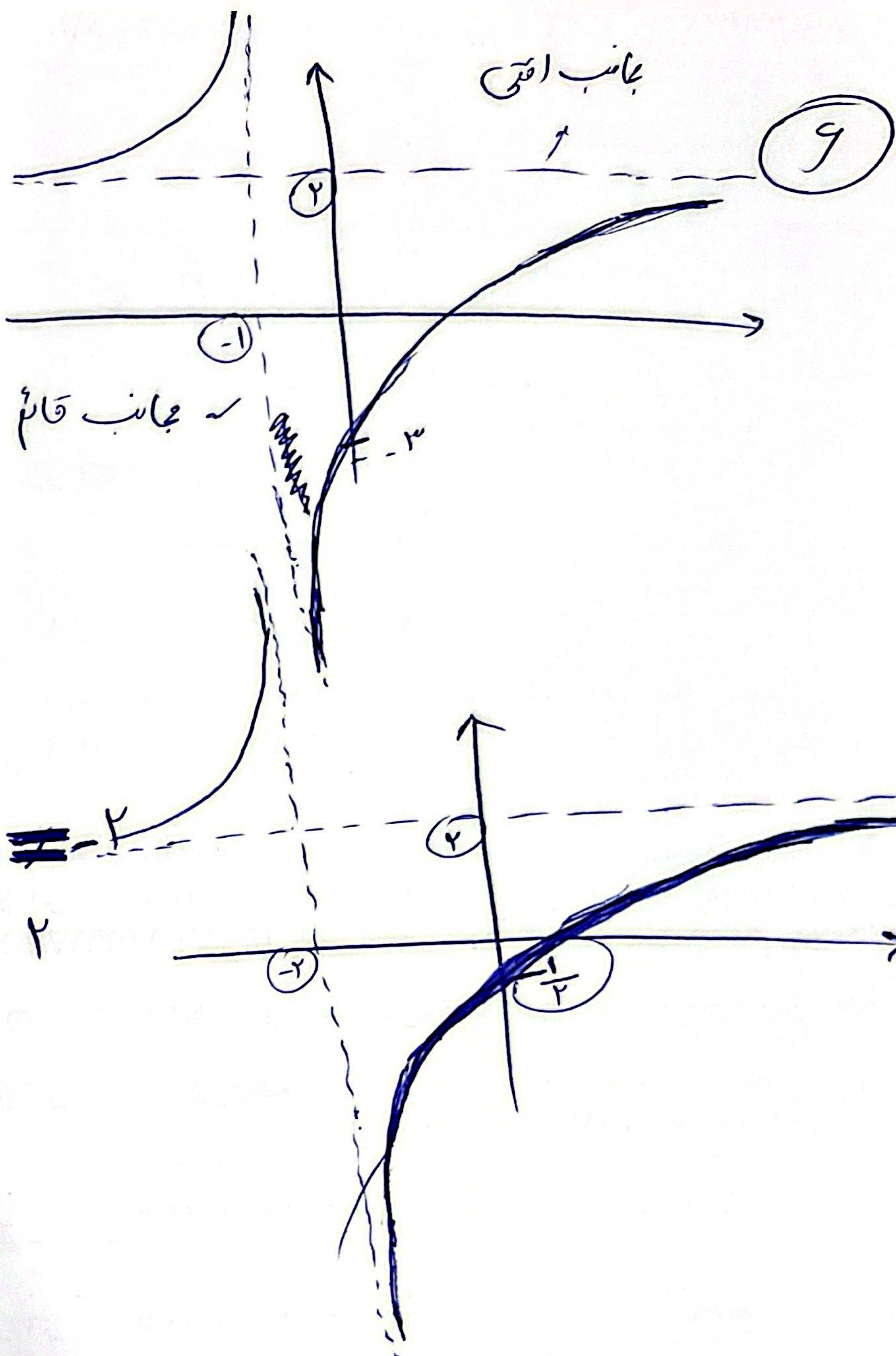
$\sqrt{\mathbb{R} - \{-2\}}$

الف) $y = \frac{2x-3}{x+1}$

شعاع $x+1=0 \rightarrow x=-1$ جانب قائم

جانب افقی $y=\frac{a}{c} \rightarrow y=2$

$x=0 \rightarrow y=3$

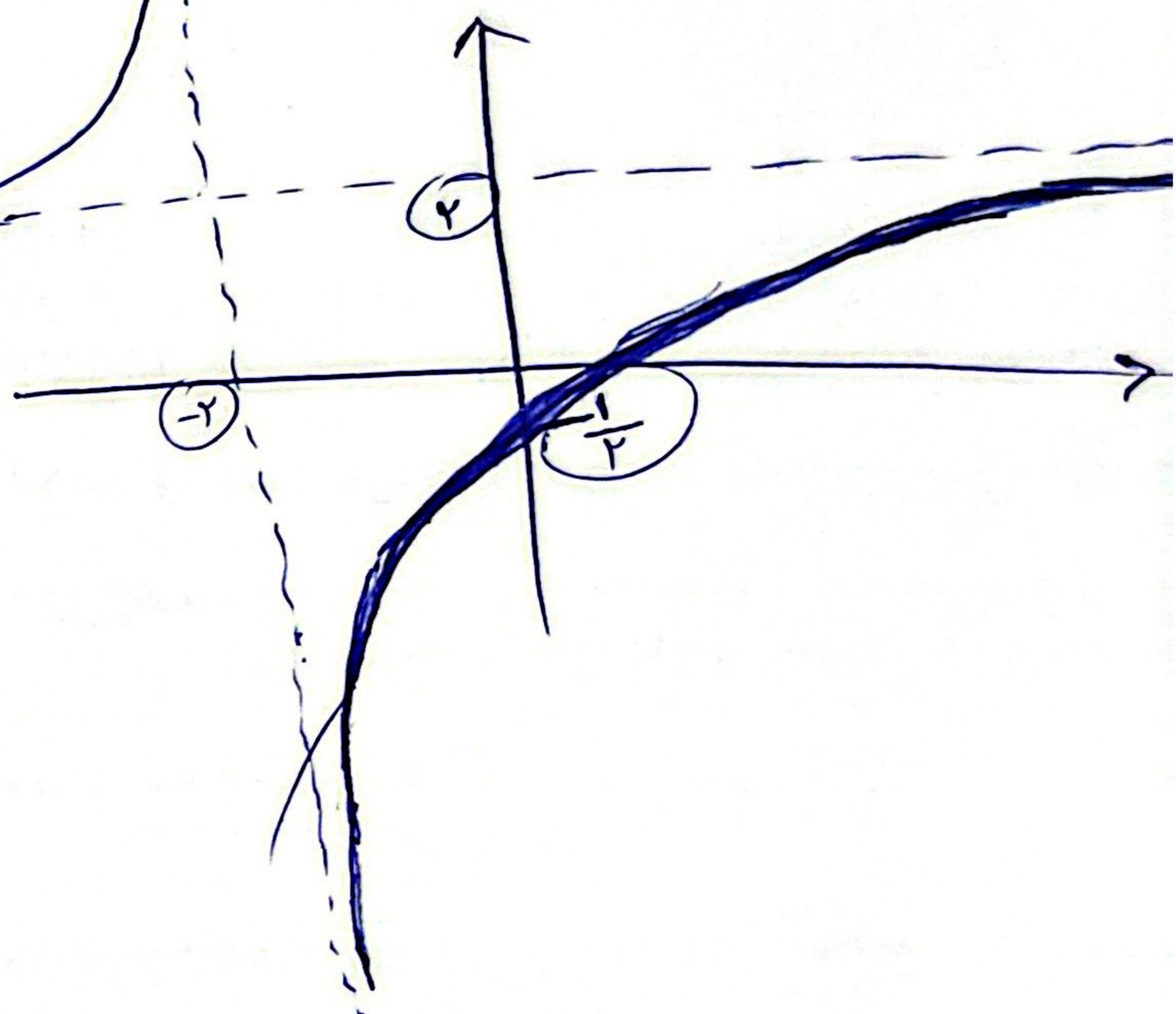


ب) $y = \frac{2x-1}{x+2}$

شعاع $x+2=0 \rightarrow x=-2$ جانب قائم

جانب افقی $y=\frac{a}{c} \rightarrow y=2$

$x=0 \rightarrow y=-\frac{1}{2}$



الف) $y = \sin x + \frac{1}{\sin x}$ $\left\{ \begin{array}{l} \alpha + \frac{1}{\alpha} \geq 2 \\ \alpha + \frac{1}{\alpha} \leq -2 \end{array} \right\} R_f = (-\infty, -2] \cup [2, +\infty)$

✓

ب) $y = \frac{x^4+1}{x^3} = x + \frac{1}{x^3} \rightarrow R_f = (-\infty, -2] \cup [2, +\infty)$

$\alpha + \frac{1}{\alpha} \geq 2$

$\alpha + \frac{1}{\alpha} \leq -2$

ج) $y = \frac{\sqrt[3]{x^2}+1}{\sqrt[3]{x}} = \sqrt[3]{x} + \frac{1}{\sqrt[3]{x}} \rightarrow R_f = (-\infty, -2] \cup [2, +\infty)$

$\alpha + \frac{1}{\alpha} \geq 2$

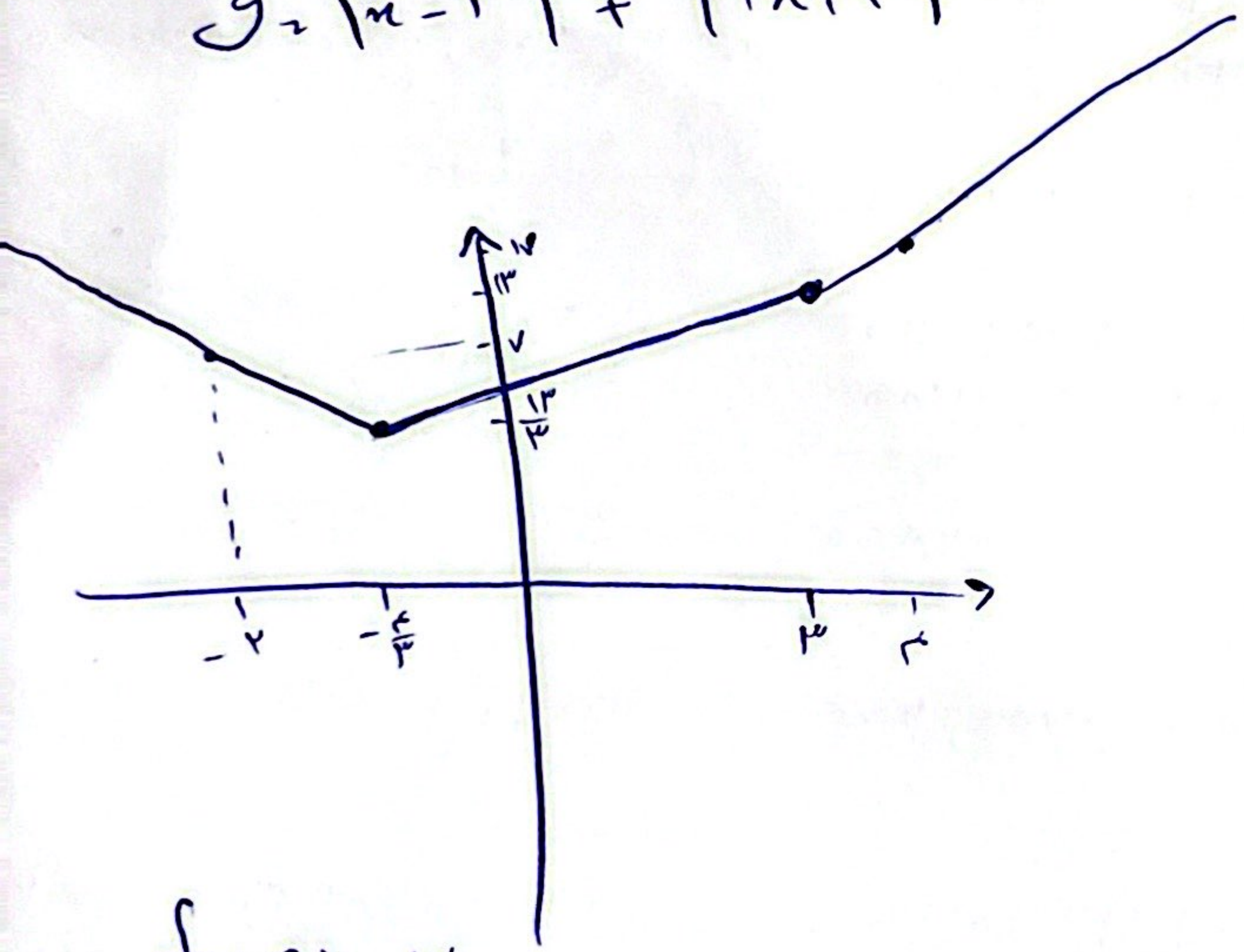
$\alpha + \frac{1}{\alpha} \leq -2$

د) $y = \sqrt{x} + \frac{1}{\sqrt{x}} \xrightarrow{\sqrt{x} \geq 0} \alpha + \frac{1}{\alpha} \geq 2 \Rightarrow R_f = [2, +\infty)$

$x^r + r > 0$
 $\downarrow a + \frac{1}{a} \geq r \xrightarrow{n=0} a > r$
 $y = x^r + \frac{1}{x^r} = \underbrace{x^r + r}_{\alpha} + \underbrace{\frac{1}{x^r + r}}_{\frac{1}{\alpha}} - r$
 $\rightarrow y = \frac{1}{r} \Rightarrow \left[\frac{1}{r}, +\infty \right) \xrightarrow{-r} Df = \left[\frac{1}{r}, +\infty \right)$

$\sqrt{x^r + r} > 0$
 $\downarrow a + \frac{1}{a} \geq r \xrightarrow{n=0} a > r$
 $y = \frac{x^r + r + 1}{\sqrt{x^r + r}} = \underbrace{\sqrt{x^r + r}}_a + \underbrace{\frac{1}{\sqrt{x^r + r}}}_{\frac{1}{a}}$
 $\downarrow \sqrt{r} + \frac{1}{\sqrt{r}} = \frac{a}{r}$
 $R_f = \left[\frac{a}{r}, +\infty \right)$

$y = |x - r| + |rx + r|$



$2 - r + rx + r = rx + 1$
 $\Rightarrow r - x + rx + r = rx + 1$
 $r - x - rx - r = -rx - 1$
 $\begin{cases} x \geq r \\ -\frac{r}{r} < x < r \\ x \leq -\frac{r}{r} \end{cases}$

$f(r) = 1$
 $f(r) = 1$
 $f(-\frac{r}{r}) = \frac{1}{r}$
 $f(-r) = 1$

$y = |x - r| + |rx + r|$

1 $x \geq r$	$rx \geq r$	rx	$x \geq r$
2 $-\frac{r}{r} < x < r$	$r < rx < r$	$r - x + rx + r = rx + 1$	$-1 < x < r$
3 $x \leq -\frac{r}{r}$	$-rx \geq r$	$r - x - rx - r = -rx - 1$	$x \leq -1$

$1 \cap 2 \cap 3 \rightarrow R_f = [r, +\infty)$

$y = |rx - r| - |x + 1|$

$\begin{cases} 1 & rx - r \geq -r \\ 2 & -r < rx - r < r \\ 3 & -rx + r \geq r \end{cases}$
 $1 \cap 2 \cap 3 \rightarrow R_f = [-r, +\infty)$
 $\begin{cases} rx - r - x - 1 = x - r \\ rx - r - x - 1 = -rx + 1 \\ rx - r + x + 1 = -x + r \end{cases}$
 $\begin{cases} x \geq 1 \\ -1 < x < 1 \\ x \leq -1 \end{cases}$