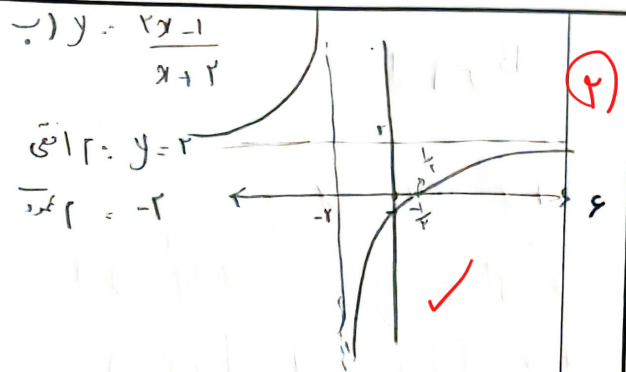
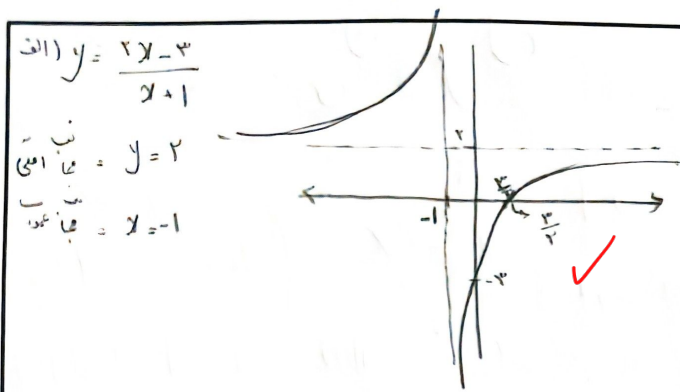


الف) $x^3 - 3x^2 + 3x = y$ $(x-1)^3 + 1 = y \rightarrow (x-1)^3 = y-1$ $x-1 = \sqrt[3]{y-1} \Rightarrow x = \sqrt[3]{y-1} + 1$ $R_f = R$ ✓	ب) $y = \frac{1}{x^2 - 2x}$ $yx^2 - 2yx - 1 = 0$ $\Delta = b^2 - 4ac \Rightarrow (-2y)^2 - 4(y)(-1) = 4y^2 + 4y = 4y(y+1)$ $y = 0 \Rightarrow$ رخ $R_f = (-\infty, -1) \cup (0, +\infty)$ ✓	۱
الف) $x^2 - 4x + 1$ $x_{\min} = \frac{4}{2} = 2$ $y_{\min} = 6$ $R_f = [6, +\infty)$ ✓	ب) $-x^2 + 4x + 3$ $x_{\max} = 3$ $y_{\max} = 11$ $R_f = (-\infty, 11]$ ✓	۱) $\sqrt{x^2 - 4x - 3}$ $x_{\min} = 2$ $y_{\min} = -7$ $R_f = [-7, +\infty)$ $\sqrt{[-7, +\infty)} = [0, +\infty)$ ✓ ۲) $\sqrt{-x^2 + 4x}$ $x_{\max} = 3$ $y_{\max} = 9$ $R_f = (-\infty, 9]$ $\sqrt{(-\infty, 9]} = [0, 3]$ ✓
الف) $x^3 - 5x^2 + 2x + 1$ $R_f = R$ ✓	ب) $y = x^3 - 4x^2 + 3x + 2$ $D_f = R$ ✓	۱) $\sqrt{x^3 - 4x^2 + 3x + 1}$ $R_f = \sqrt{R} = [0, +\infty)$ ✓ ۲) $y = (x^3 - 4x^2 + 3x + 1)^2$ $R_f = (R)^2 = [0, +\infty)$ ✓
الف) $y = \frac{3x+1}{x-2} \Rightarrow R_f = R - \{2\}$ ✓ $R_f = R - \{2\}$	ب) $y = \frac{2x+1}{x+1}$ $R_f = R - \{1\}$ ✓	۲
الف) $y = \sqrt{\frac{4x+4}{x+1}} \rightarrow R_f = R - \{1\}$ $\sqrt{R - \{1\}} \Rightarrow [0, \sqrt{2}) \cup (\sqrt{2}, +\infty)$ ✓	ب) $y = \sqrt{\frac{3x+1}{2-x}} \rightarrow R_f = R - \{-2\}$ $\sqrt{R - \{-2\}} \Rightarrow [0, +\infty)$ ✓	۳



الف) $y = \sin x \cdot \frac{1}{\sin x}$

$R_f = (-\infty, -\pi) \cup (\pi, +\infty)$ ✓

ب) $y = \frac{x^4+1}{x^3} = x + \frac{1}{x^3}$

$R_f = (-\infty, -\pi) \cup (\pi, +\infty)$ ✓

ج) $y = \frac{\sqrt[3]{x^2+1}}{\sqrt[3]{x}}$

$R_f = (-\infty, \pi] \cup (\pi, +\infty)$ ✓

د) $y = \sqrt{x} + \frac{1}{\sqrt{x}}$

$R_f = [\pi, +\infty)$ ✓

الف) $y = x^2 + \frac{1}{x^2+3}$

$x^2_{\min} = 0 \rightarrow R_f = [\frac{1}{3}, +\infty)$ ✓

ب) $y = \frac{x^2+\delta}{\sqrt{x^2+\epsilon}} = \frac{x^2+\epsilon+1}{\sqrt{x^2+\epsilon}}$

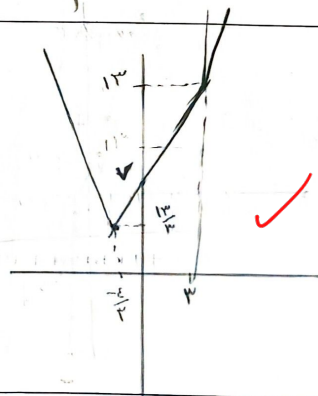
$\sqrt{x^2+\epsilon} + \frac{1}{\sqrt{x^2+\epsilon}} \rightarrow \min \sqrt{0^2+\epsilon} + \frac{1}{\sqrt{0^2+\epsilon}} = \frac{\delta}{\epsilon}$

$R = [\frac{\delta}{\epsilon}, +\infty)$ ✓

$y = |x-3| + |3x+1|$

$-\frac{\epsilon}{3}$	3	
$3-x-\epsilon-3x$	$3-x+3x+\epsilon$	$x-3+3x+\epsilon$
$(-4x-1)$	$(2x+\epsilon)$	$(4x+1)$

if $\begin{cases} x \geq 3 \rightarrow \epsilon x + 1 \\ -\frac{\epsilon}{3} \leq x < 3 \rightarrow 2x + \epsilon \\ x < -\frac{\epsilon}{3} \rightarrow -4x - 1 \end{cases}$

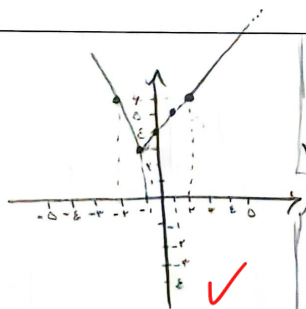


الف) $y = |x-1| + |2x+1|$

-1	1	
$x-x-1-2x$	$x-x+1+2x$	$x-1+2x+1$
$(-3x-1)$	$(x+1)$	$(3x)$

if $\begin{cases} x \geq 1 \rightarrow 3x \\ -1 \leq x < 1 \rightarrow x+1 \\ x < -1 \rightarrow -3x-1 \end{cases}$

$R_f = [\pi, +\infty)$



ب) $y = |2x-1| - |x+1|$

-1	1	
$2-x-x+1$	$2-x-1-1$	$2x-1-x-1$
$(-x+1)$	$(-x+1)$	$(x-2)$

if $\begin{cases} x \geq 1 \rightarrow x-2 \\ -1 \leq x < 1 \rightarrow -x+1 \\ x < -1 \rightarrow -x+1 \end{cases}$

$R_f = [-\pi, +\infty)$

