

الف) $y = x^3 - 3x^2 + 3x \Rightarrow y = x^3 - 3x^2 + 3x - 1 + 1 \Rightarrow y = (x-1)^3 + 1$
 $\Rightarrow y-1 = (x-1)^3 \Rightarrow \sqrt[3]{y-1} = x-1 \Rightarrow x = \sqrt[3]{y-1} + 1 \Rightarrow R_f = \mathbb{R}$
 ب) $y = \frac{1}{x^2 - 2x} \Rightarrow y = \frac{1}{x(x-2)} \Rightarrow x=0 \vee x=2 \Rightarrow yx^2 - 2yx = 1$
 $\Rightarrow x(yx - 2y) = 1 \Rightarrow yx - 2y = \frac{1}{x} \Rightarrow yx - \frac{1}{x} = 2y \Rightarrow x=0 \Rightarrow \frac{1}{x} = \frac{1}{0} = \infty$
 $\Rightarrow x=2 \Rightarrow 2y - \frac{1}{2} = 2y \Rightarrow -\frac{1}{2} \neq 0 \Rightarrow R_f = \mathbb{R} - \{0, 2\}$

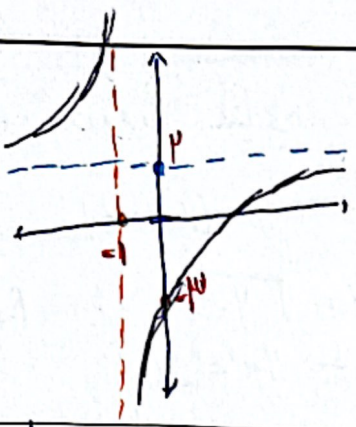
الف) $y = x^2 - 4x + 10 \Rightarrow a > 0 \Rightarrow x_{min} = -\frac{b}{2a} = 2 \Rightarrow y_{min} = 6 \Rightarrow R_f = [6, +\infty)$
 ب) $y = -x^2 + 4x + 3 \Rightarrow a < 0 \Rightarrow x_{max} = -\frac{b}{2a} = 2 \Rightarrow y_{max} = 11 \Rightarrow R_f = (-\infty, 11]$
 ۲) $y = \sqrt{x^2 - 4x + 3} \Rightarrow a > 0 \Rightarrow x_{min} = -\frac{b}{2a} = 2 \Rightarrow y_{min} = -1 \Rightarrow R_f = [-1, +\infty) \Rightarrow [0, +\infty)$
 ۳) $y = \sqrt{4x - x^2} \Rightarrow a < 0 \Rightarrow x_{max} = -\frac{b}{2a} = 2 \Rightarrow x = 3 \Rightarrow y_{max} = 2 \Rightarrow R_f = [-\infty, 2] \Rightarrow [0, 2]$

الف) $y = x^3 - 2x^2 + 2x + 1 \Rightarrow R_f = \mathbb{R}$
 ب) $y = x^4 - 4x^2 + 3 \Rightarrow R_f = \mathbb{R}$
 ۲) $y = \sqrt{x^4 - 4x^2 + 3} \Rightarrow R_f = [0, +\infty)$
 ۳) $y = (x^2 - 4x + 3)^2 \Rightarrow R_f = [0, +\infty)$
 چون توان ۲ است به ازای هر مقدار x عددی مثبت در اختیار داریم پس داریم $R_f = [0, +\infty)$

الف) $y = \frac{3x+1}{x-2} \Rightarrow R_f = \mathbb{R} - \{3\}$
 ب) $y = \frac{2x+1}{x+1} \Rightarrow R_f = \mathbb{R} - \{2\}$

الف) $y = \sqrt{\frac{x+1}{x-1}} \Rightarrow y^2 = \frac{x+1}{x-1} \Rightarrow y^2x + y^2 = x+1 \Rightarrow y^2x - x = 1 - y^2$
 $\Rightarrow x(y^2 - 1) = -y^2 + 1 \Rightarrow x = \frac{1 - y^2}{y^2 - 1} \Rightarrow R_f = \mathbb{R} - \{\pm\sqrt{2}\} \Rightarrow R_f = [0, +\infty) - \{\sqrt{2}\}$
 ب) $y = \sqrt{\frac{3x+1}{2-x}} \Rightarrow y^2 = \frac{3x+1}{2-x} \Rightarrow 2y^2 - y^2x = 3x+1 \Rightarrow 3x + y^2x = 2y^2 - 1$
 $\Rightarrow x(3 + y^2) = 2y^2 - 1 \Rightarrow x = \frac{2y^2 - 1}{3 + y^2} \Rightarrow R_f = \mathbb{R} \Rightarrow R_f = [0, +\infty)$
 مقدار مثبت

- ① $\frac{y-3}{x+1}$ (الف)
- ② $\frac{y-3}{x+1} = -1$ (ب)
- ③ $\frac{y-3}{x+1} = 2$ (ج)
- ④ $x=0 \Rightarrow y = -3$ (د)

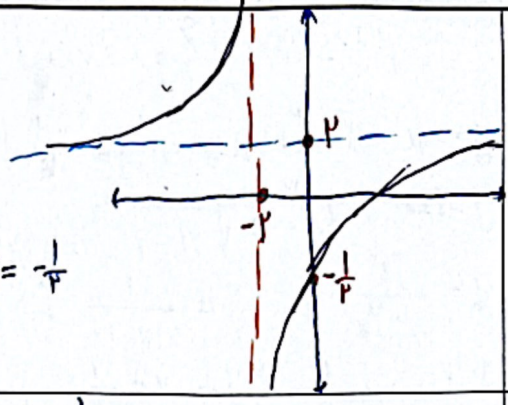


ب) $y = \frac{x-1}{x+2}$

① $x = -2$

② $x = 2$

③ $x=0 \Rightarrow y = -\frac{1}{2}$



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الف) $\sin x + \frac{1}{\sin x} = y \Rightarrow R_f = (-\infty, -2] \cup [2, +\infty)$

ب) $y = \frac{x^4+1}{x^2} \Rightarrow x^2 + \frac{1}{x^2} \Rightarrow R_f = (-\infty, 2] \cup [2, +\infty)$

ج) $y = \frac{\sqrt[3]{x^2}+1}{\sqrt[3]{x}} \Rightarrow y = \sqrt[3]{x} + \frac{1}{\sqrt[3]{x}} \Rightarrow R_f = (-\infty, -2] \cup [2, +\infty)$

د) $y = \sqrt{x} + \frac{1}{\sqrt{x}} \Rightarrow R_f = [2, +\infty)$

الف) $y = x^2 + \frac{1}{x^2+3} \Rightarrow x=0 \Rightarrow y = \frac{1}{3} \Rightarrow$ این‌ها را می‌توانیم با استفاده از مشتق به دست آوریم
 $\Rightarrow R_f = [\frac{1}{3}, +\infty)$

ب) $y = \frac{x^2+4}{\sqrt{x^2+4}} \Rightarrow \frac{x^2+4}{\sqrt{x^2+4}} \Rightarrow \sqrt{x^2+4} + \frac{1}{\sqrt{x^2+4}}$
 $\Rightarrow R_f = [\frac{2}{\sqrt{5}}, +\infty)$

$y = |x-3| + |3x+4|$

$x=0 \Rightarrow y=7$

$x=3 \Rightarrow y=13$

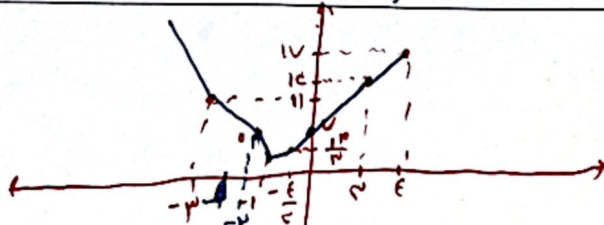
$x=-\frac{4}{3} \Rightarrow y=\frac{13}{3}$

$x=4 \Rightarrow y=17$

$x=-1 \Rightarrow y=5$

$x=-2 \Rightarrow y=7$

$x=-3 \Rightarrow y=11$



این‌ها را می‌توانیم با استفاده از مشتق به دست آوریم

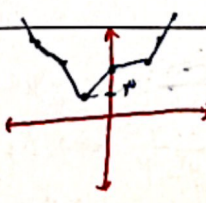
الف) $y = |x-3| + |2x+1| \Rightarrow x=3, x=-1 \Rightarrow$

$y = \begin{cases} x-3+2x+1 & x > 3 \\ 3-x+2x+1 & -1 \leq x \leq 3 \\ 3-x-2x-1 & x < -1 \end{cases} \Rightarrow \begin{cases} 3x-2 & x > 3 \\ x+4 & -1 \leq x \leq 3 \\ -3x & x < -1 \end{cases}$

ب) $y = |2x-1| - |x+1|$

$x=1, x=-1 \Rightarrow$

$y = \begin{cases} 2x-1-x-1 & x > 1 \\ -2x+1-x-1 & -1 \leq x \leq 1 \\ -2x+1+x+1 & x < -1 \end{cases} \Rightarrow \begin{cases} x-2 & x > 1 \\ -3x & -1 \leq x \leq 1 \\ -x+2 & x < -1 \end{cases}$



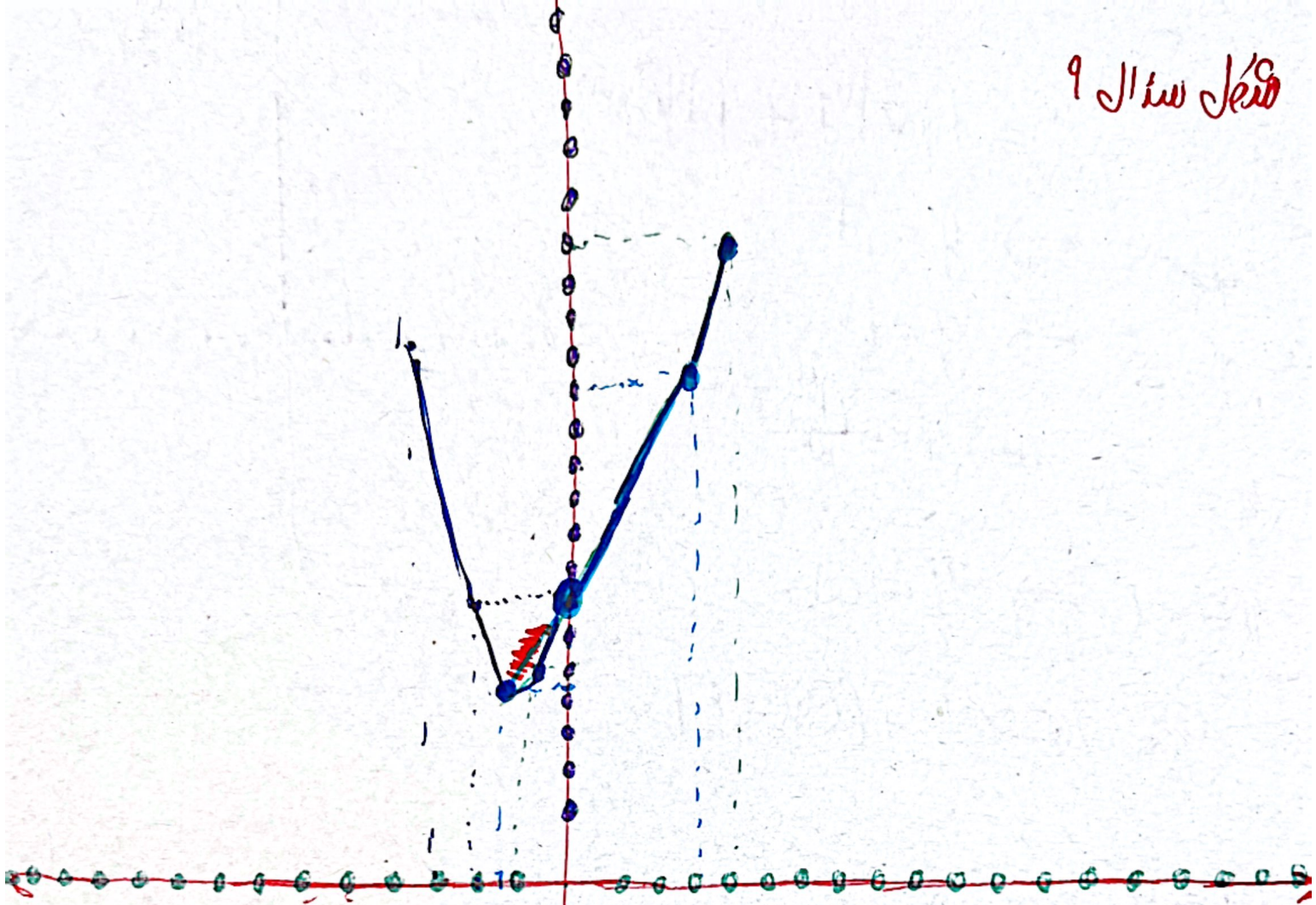
این‌ها را می‌توانیم با استفاده از مشتق به دست آوریم



$R_f = [-2, +\infty)$

این‌ها را می‌توانیم با استفاده از مشتق به دست آوریم

سوال 9



$$\frac{u = -c \Rightarrow 11}{u = -r \Rightarrow 17}$$

$$\frac{a = -1 \Rightarrow y = \omega}{y}$$

$$y = |x - \omega| + |y \omega + \epsilon|$$

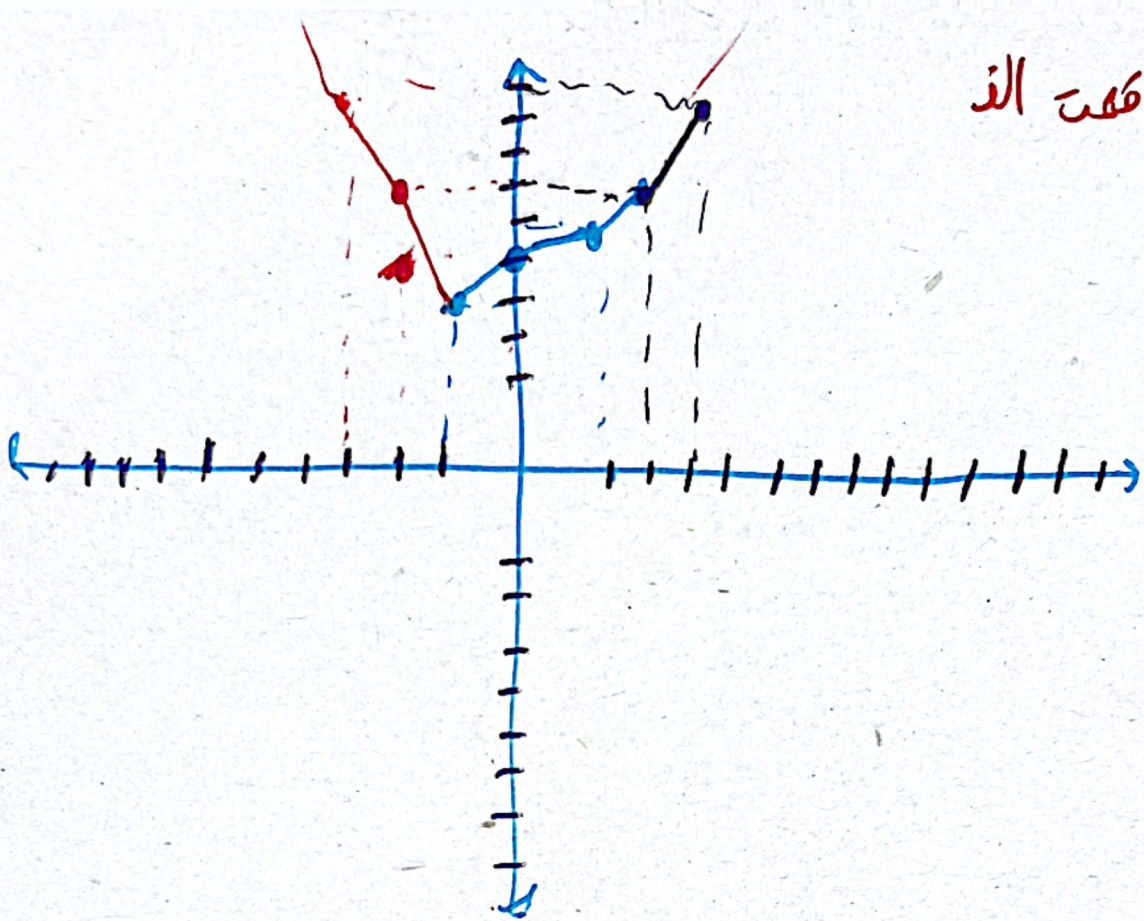
$$\frac{a = 0 \Rightarrow y = 17}{y}$$

$$\frac{a = \omega \Rightarrow y = 17}{y}$$

$$\frac{x = \sum_{i=1}^n \Rightarrow y = 17}{y}$$

$$\frac{a = \epsilon \Rightarrow y = 17}{y}$$

سؤال ما بعد الذ



$$x = 2 \Rightarrow y = 4$$

$$x = 1 \Rightarrow y = 3$$

$$x = 0 \Rightarrow y = 2$$

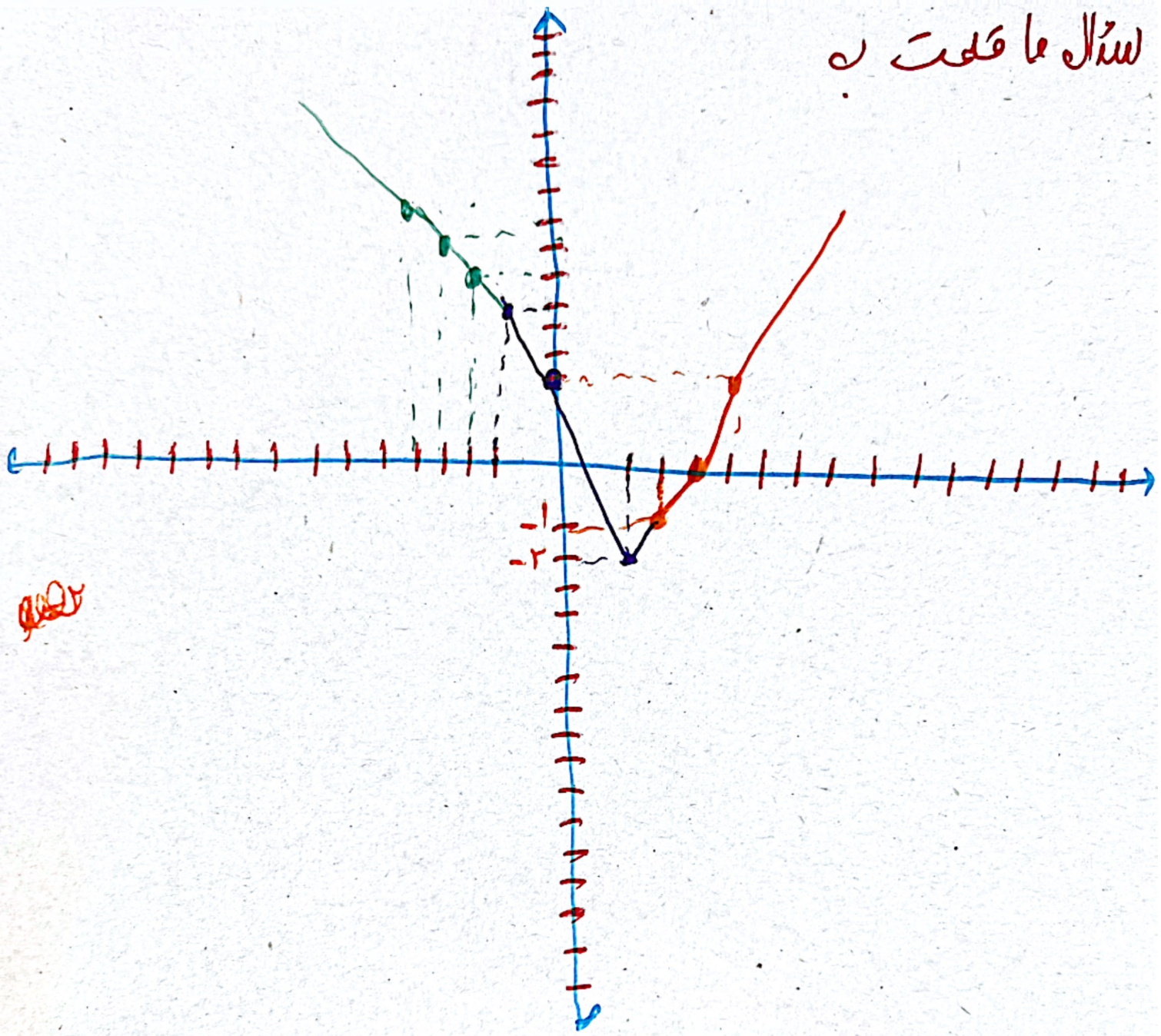
$$x = 1 \Rightarrow y = 0$$

$$x = 2 \Rightarrow y = 2$$

$$x = -2 \Rightarrow y = 4$$

$$x = -2 \Rightarrow y = 9$$

سؤال ما قلعت ب



القطعة

$$x=2 \Rightarrow y=-1$$

$$x=3 \Rightarrow y=0$$

$$x=4 \Rightarrow y=1$$

$$x=-1 \Rightarrow y=2$$

$$x=0 \Rightarrow y=1$$

$$x=1 \Rightarrow y=-2$$

$$x=-2 \Rightarrow y=2$$

$$x=-3 \Rightarrow y=1$$

$$x=-4 \Rightarrow y=1$$