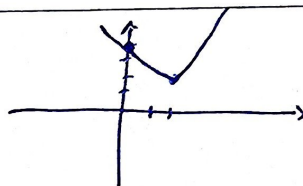


الف) $y = x + |2x - 4|$

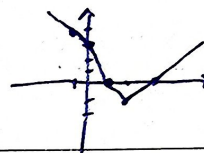
$$\begin{array}{c|c} x-2x+4 & x+2x-4 \\ \hline -x+4 & 3x-4 \end{array}$$



۱

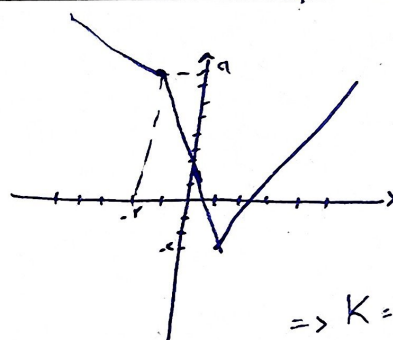
ب) $y = |x-2| + |x-1| - |x|$

$$\begin{aligned} x=3 &\Rightarrow y=0 \\ x=2 &\Rightarrow y=-1 \\ x=1 &\Rightarrow y=0 \\ x=0 &\Rightarrow y=3 \\ x=-1 &\Rightarrow y=4 \end{aligned}$$



$|3x-3| - |x+2| = K$

$$\begin{array}{c|c|c} -2 & 1 & \\ \hline -2x+3+x+2 & -2x+3-x-2 & 3x-3-x-2 \\ \hline -x+5 & -x+1 & 2x-5 \end{array}$$

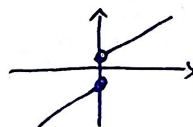


$\Rightarrow K = -3$

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الف) $y = x + \frac{x}{|x|}$

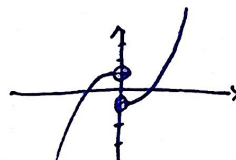
$$\begin{array}{c|c} 0 & \\ \hline x-1 & x+1 \end{array}$$



۳

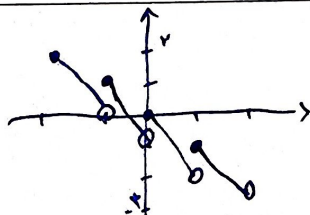
ب) $y = x|x| - \frac{x}{|x|}$

$$\begin{array}{c|c} 0 & \\ \hline -x^2+1 & x^2-1 \end{array}$$



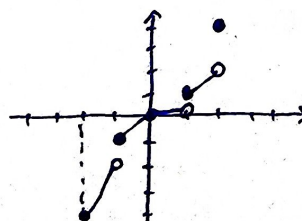
الف) $y = [x] - 2x$

$x=2 \Rightarrow y=-2$



ب) $y = [x] |x|$

$x=2 \Rightarrow y=4$



۴

الف) $y = 3x - [2x] \xrightarrow{a=2x} y = a - [a] \Rightarrow R_y = [0, 1)$

ب) $y = 4x - 4[x] \Rightarrow 4(x - [x]) \Rightarrow R_y = [0, 4)$

ج) $y = x - 5[\frac{x}{5}] = 5(\frac{x}{5} - [\frac{x}{5}]) \Rightarrow R_y = [0, 5)$

د) $y = [2x] - 2x \Rightarrow y = [a] - a \Rightarrow R_y = (-1, 0]$

۵

الف) $y = 3 \sin x + 2 \cos x$

$$\left. \begin{aligned} -\sqrt{13} &\leq 3 \sin x + 2 \cos x \leq \sqrt{13} \end{aligned} \right\} \Rightarrow Ry = [-\sqrt{13}, \sqrt{13}] \Rightarrow -\sqrt{a^2+b^2} \leq a \sin x + b \cos x \leq \sqrt{a^2+b^2}$$

ب) $y = 2 \sin x - 3 \cos x \Rightarrow -5 \leq 2 \sin x - 3 \cos x \leq 5 \Rightarrow Ry = [-5, 5]$

ج) $y = [2 \sin x + 5 \cos x] \Rightarrow -\sqrt{29} \leq 2 \sin x + 5 \cos x \leq \sqrt{29} \Rightarrow Ry = [-\sqrt{29}, \sqrt{29}]$

د) $y = \sqrt{\cos x - 2 \sin x} \Rightarrow -\sqrt{5} \leq \cos x - 2 \sin x \leq \sqrt{5} \Rightarrow Ry = [0, \sqrt{5}]$

هـ) $y = \sin^2 x + 3 \sin x + 2$
 $\sin x = 1 \Rightarrow y = 6$
 $\sin x = -1 \Rightarrow y = 0 \Rightarrow Ry = [0, 6]$
 $\sin x = \frac{-b}{2a} = -\frac{3}{2}$ قوع

و) $y = 2 \sin^2 x + 3 \sin x + 2$
 $\sin x = 1 \Rightarrow y = 7$
 $\sin x = -1 \Rightarrow y = 1 \Rightarrow Ry = [\frac{1}{2}, 7]$
 $\sin x = \frac{-b}{2a} = -\frac{3}{4} \Rightarrow y = \frac{1}{2}$

الف) $y = \frac{\sin^2 x}{1 - \cos^2 x} = \frac{\sin^2 x}{\sin^2 x} = 1$
 $\cos x = 1 \Rightarrow y = 1$
 $\cos x = -1 \Rightarrow y = 1 \Rightarrow Ry = [1, 1]$

ب) $y = \frac{2 \cot x}{1 + \cot^2 x} = \frac{2 \cot x}{\frac{1}{\sin^2 x}} = 2 \cos x \sin x = \sin 2x$
 $Ry = [-1, 1]$

الف) $y = \sin^2 x + \cos^2 x \Rightarrow \frac{1}{2} \leq \sin^2 x + \cos^2 x \leq 1 \Rightarrow Ry = [\frac{1}{2}, 1]$

ب) $y = [\sin^2 x + \cos^2 x] \Rightarrow \frac{1}{2} \leq \sin^2 x + \cos^2 x \leq 1 \Rightarrow Ry = \{0, 1\}$
 اعمال برکت

الف) $y = \frac{2x^2 + 7x - 4}{x+1} = \frac{(x+1)(2x+1)}{x+1} = 2x+1$

$Dy = \mathbb{R} - \{-1\} \Rightarrow Ry = \mathbb{R} - \{-1\}$

ب) $y = \frac{x^2 - 1}{x-1} = \frac{(x-1)(x+1)}{x-1} = x+1$

$2(-1) - 1 = -3$

$\frac{-b}{2a} = -\frac{1}{2}$

$y_{\min} = \frac{3}{2}$

$Ry = [\frac{3}{2}, +\infty)$