۱۸

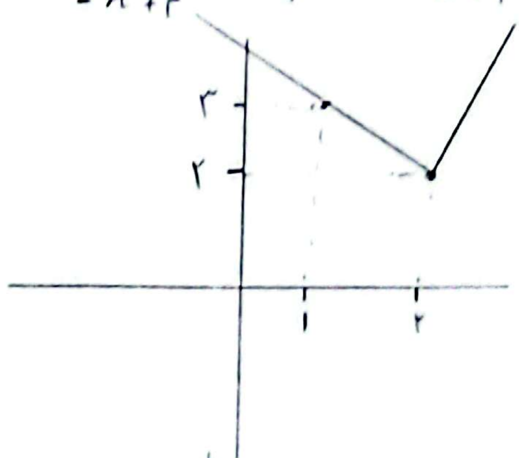
۱-

الف) $y = x + |2x - 4|$

	۲	
$\underbrace{x - 2x + 4}_{-x+4}$		$\underbrace{x + 2x - 4}_{3x - 4}$

۲

$$\begin{cases} 3x - 4 & x > 2 \\ 2 & x = 2 \\ -x + 4 & x < 2 \end{cases}$$

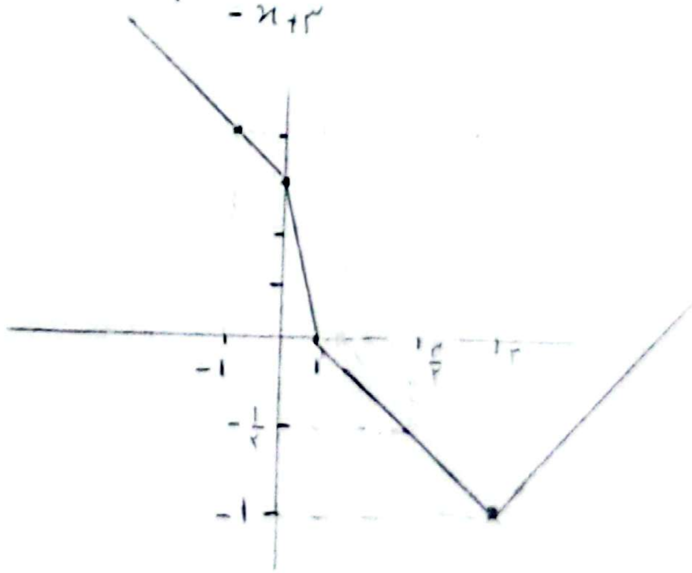


$R_f = [2, +\infty)$ ✓

ب) $y = |x-2| + |x-1| - |x|$

	0		1		2	
$\underbrace{-x+2-x+1+x}_{-x+3}$		$\underbrace{-x+2-x+1-x}_{-3x+3}$		$\underbrace{-x+2+x-1-x}_{-x+1}$		$\underbrace{x-2+x-1-x}_{x-3}$

$$\begin{cases} -x + 3 & x < 0 \\ -3x + 3 & 0 \le x \le 1 \\ -x + 1 & 1 < x < 2 \\ x - 3 & x \ge 2 \end{cases}$$



$R_f = [-1, +\infty)$ ✓

۲- ۲

$|3x - 3| - |x + 2| = k \longrightarrow 3x - 3 = 0 \longrightarrow x = 1$

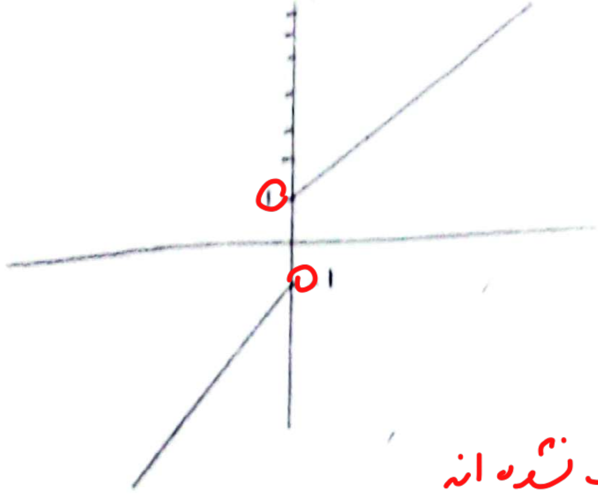
$x + 2 = 0 \longrightarrow x = -2$

$$\begin{cases} \text{if } x < -2 \longrightarrow -3x + 3 - (-(x+2)) = -2x + 5 \longrightarrow (9, +\infty) \\ \text{if } -2 \le x < 1 \longrightarrow -3x + 3 - x - 2 = -4x + 1 \longrightarrow [-3, 9] \\ \text{if } x \ge 1 \longrightarrow 3x - 3 - x - 2 = 2x - 5 \longrightarrow [-3, +\infty) \end{cases}$$

$k = -3, 9$ ✓

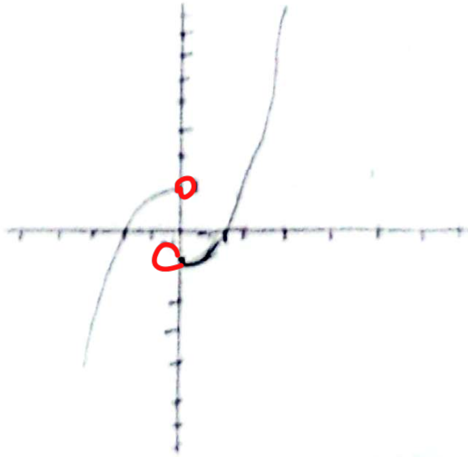
۳- ۱,۵

الف) $x + \frac{x}{|x|}$



صفرها تعریف نشده اند

ب) $y = x|x| - \frac{x}{|x|}$

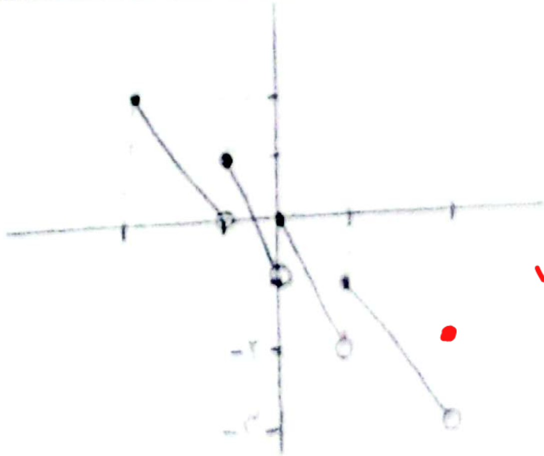


$[-2,2]$

۴- ۱

الف) $y = [x] - 2x$

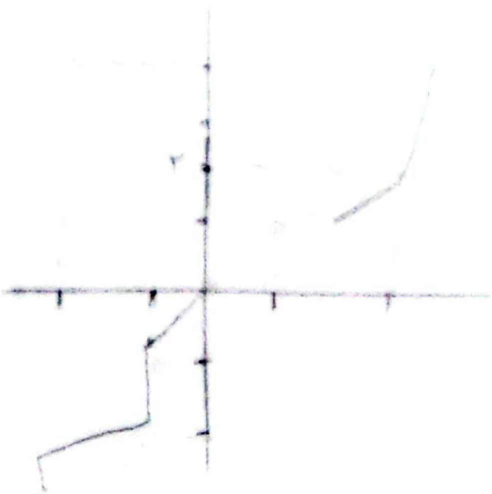
$$\begin{cases} -2 \le x < -1 \rightarrow -2x-2 \\ -1 \le x < 0 \rightarrow -2x-1 \\ 0 \le x < 1 \rightarrow -2x \\ 1 \le x < 2 \rightarrow -2x+1 \end{cases}$$



✓

انتهای بازه رو هم باید مشخص کنی

ب) $y = [x]|x|$



این چیه ؟!

۱/۷۵

الف) $y=3x-[3x] \longrightarrow R_f=[0,1)$ ✓

ب) $y=4x-4[x] \longrightarrow y=4(x-[x]) \longrightarrow R_f=[0,4)$ ✓

ج) $y=x-5[\frac{x}{5}] \to y=5(\frac{x}{5}-[\frac{x}{5}]) \to R_f=[0,5)$ ✓

د) $y=[2x]-2x \longrightarrow R_f=(-1,0]$ دقت!

الف) $y=3\sin x+2\cos x \longrightarrow y^2=(3\sin x+2\cos x)^2=9\sin^2 x+12\sin x\cos x+4\cos^2 x$ -۶

$y^2=4+5\sin^2 x+\underbrace{12\sin x\cos x}_{6\sin 2x} \longrightarrow -1\le \sin x,\cos x\le +1 \to [-\sqrt{13},\sqrt{13}]=R_f$ ✓

ب) $y=4\sin x-3\cos x \longrightarrow -\sqrt{16+9}\le 4\sin x-3\cos x\le \sqrt{16+9}$ $\quad R_f=[-5,5]$ ✓

ج) $y=[2\sin x+5\cos x]$

۱/۷۵

$-\sqrt{29}\le 2\sin x+5\cos x\le \sqrt{29} \longrightarrow [2\sin x+5\cos x]=\{-6,-5,-4,-3,-2,\dots,5\}$

$R_f=\{-6,-5,-4,-3,-2,-1,0,1,2,3,4,5\}$ ✓

د) $\sqrt{\cos x-2\sin x} \to \quad x-2\sin x\ge 0 \to \cos x\ge 2\sin x \to \frac{\cos}{\sin}\ge 2 \to \cot x\ge 2$

$\ge 0 \to 0\le f(x)\le\sqrt{\sqrt{5}} \to f(x)\in[0,\sqrt{5}]$

$R_f=[0,\sqrt[4]{5}]$

دوبار رادیکال نزدی!

الف) $\sin^2 x+3\sin x+2 \qquad -1\le y\le 1 \to y^2+3y+2 \to y_{min}=\frac{-b}{2a}\to -1.5$ -۷

۲

راس $\to (-1.5)^2+3(-1.5)+2=-0.25 \qquad y=-1\to 1-3+2=0,\ y=1\to 1+3+2=6$

✓ $R_f=[0,6]$

ب) $2\sin^2 x+3\sin x+2 \longrightarrow -1\le y\le 1 \to 2y^2+3y+2 \to y_{min}=-\frac{b}{2a}=-0.75$

راس $\to 0.875,\ y=-1\to 1,\ y=1\to 7 \to R_f=[0.875,7]$ ✓

-۸

الف) $\sin^2 x + 4\cos^2 x \to (1-\cos^2 x) + 4\cos^2 x = 1+3\cos^2 x \to 0 \le \cos^2 x \le 1$

$\cos^2 x = 0 \to 1+3\cdot 0 = 1$ و $\cos^2 x = 1 \to 1+3\cdot 1 = 4 \to R_f = [1,4]$ ✓ ۲

ب) $\frac{2\cot x}{1+\cot^2 x} \to \frac{2\frac{\cos}{\sin}}{1+\frac{\cos^2}{\sin^2}} \to \frac{2\frac{\cos}{\sin}}{\frac{1}{\sin^2}} = \frac{2(\cos)(\cancel{\sin^2})^{\sin}}{\cancel{\sin}} = 2\cos\sin$

✓ $R_f = [-1,1]$

-۹

الف) $\sin^6 x + \cos^6 x \Rightarrow y_{min} = -\frac{b}{2a} \to \frac{1}{2} \to$ راس $= \frac{1}{4} \to (\sin^2 x + \cos^2 x)^3 = 1$

$\to R_f = [\frac{1}{4}, 1]$ ✓ ۲

ب) $[\sin^8 x + \cos^8 x] \to (\sin^2 x)^4 + (\cos^2 x)^4$

$\to 1-\sin^2 x \to \sin^2 x \in [0,1]$

$f(y) = 2y^4 - 4y^3 + 6y^2 - 4y + 1 \to f(0) = 1,\ f(1) = 1$ $\quad R_f = \{0,1\}$ ✓

-۱۰ ۲

الف) $\frac{2x^2+7x-4}{x+4} \to \frac{2x^2+8x-x-4}{x+4} = \frac{(2x^2+8x)-(x+4)}{x+4}$

$f(x) = \frac{2x(x+4)-(x+4)}{x+4} \to \frac{(2x-1)(x+4)}{x+4} \to f(x) = 2x-1 \to$

✓ $(-\infty,-9)\cup(-9,+\infty)$

ب) $\frac{x^3-1}{x-1} \to \frac{\cancel{(x-1)}(x^2+1+x)}{\cancel{x-1}} = x^2+x+1 = 0 \xrightarrow{x\neq 1}$ راس $x = \frac{-b}{2a} = -0/5$

راس $\to 0/75$ و $f(1) = 3 \to [0/75, +\infty)$ ✓