مدرس نمر اصفهانی

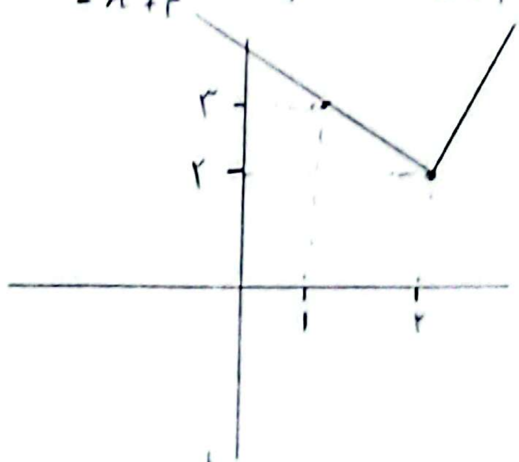
-۱

الف) $y = x + |2x-4|$

$$\begin{array}{c|c|c} & 2 & \\ \hline & \underbrace{x-2x+4}_{-x+4} & \underbrace{x+2x-4}_{3x-4} \end{array}$$

$$\begin{cases} 3x-4 & x>2 \\ 2 & x=2 \\ -x+4 & x<2 \end{cases}$$

$R_f = [2, +\infty)$

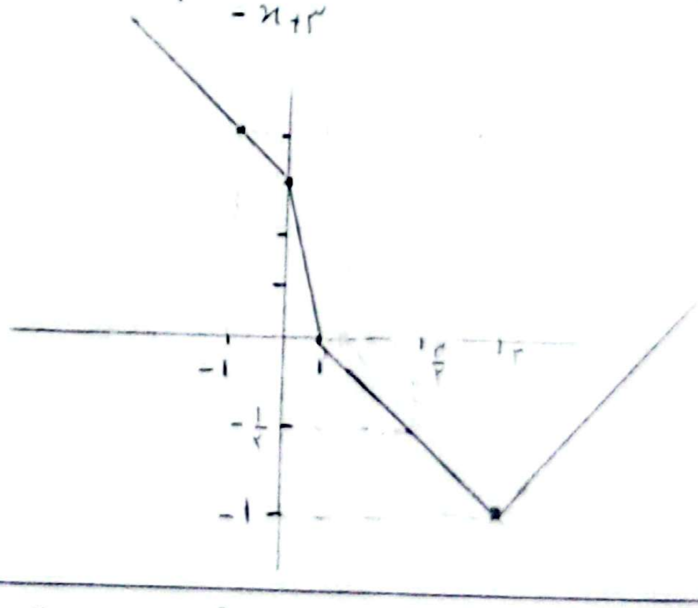


ب) $y = |x-2| + |x-1| - |x|$

$$\begin{array}{c|c|c|c} & 0 & 1 & 2 \\ \hline \underbrace{-x+2-x+1+x}_{-x+3} & \underbrace{-x+2-x+1-x}_{-3x+3} & \underbrace{-x+2+x-1-x}_{-x+1} & \underbrace{x-2+x-1-x}_{x-3} \end{array}$$

$$\begin{cases} -x+3 & x<0 \\ -3x+3 & 0 \le x \le 1 \\ -x+1 & 1<x<2 \\ x-3 & x \ge 2 \end{cases}$$

$R_f = [-1, +\infty)$



-۲

$|3x-3| - |x+2| = k \longrightarrow 3x-3=0 \longrightarrow x=1$

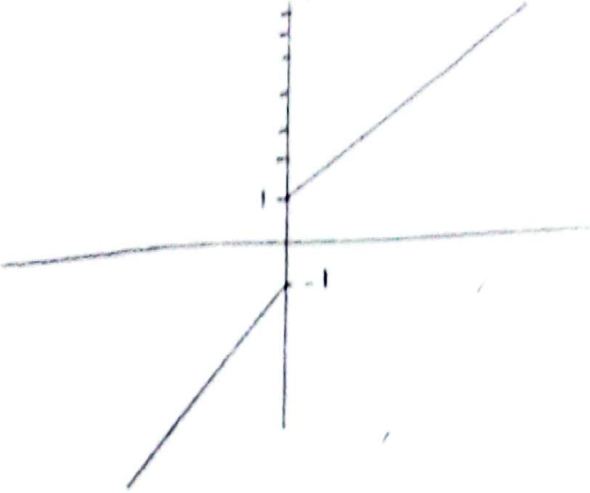
$x+2=0 \longrightarrow x=-2$

$$\begin{cases} \text{if } x<-2 \longrightarrow -3x+3-(-(x+2)) = -2x+5 \longrightarrow (9, +\infty) \\ \text{if } -2 \le x < 1 \longrightarrow -3x+3-x-2 = -4x+1 \longrightarrow [-3, 9] \\ \text{if } x \ge 1 \longrightarrow 3x-3-x-2 = 2x-5 \longrightarrow [-3, +\infty) \end{cases}$$

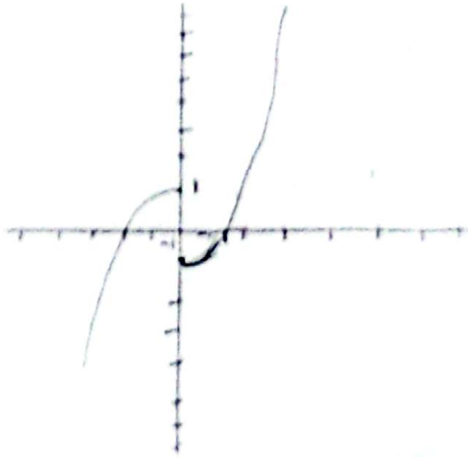
$k = -3, \cancel{9}$

-۳

الف) $x + \frac{x}{|x|}$



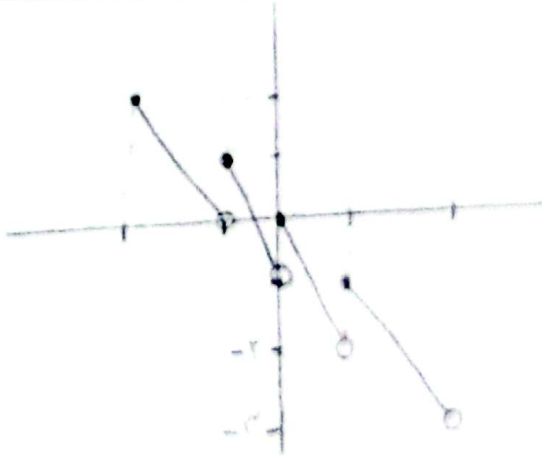
ب) $y = x|x| - \frac{x}{|x|}$



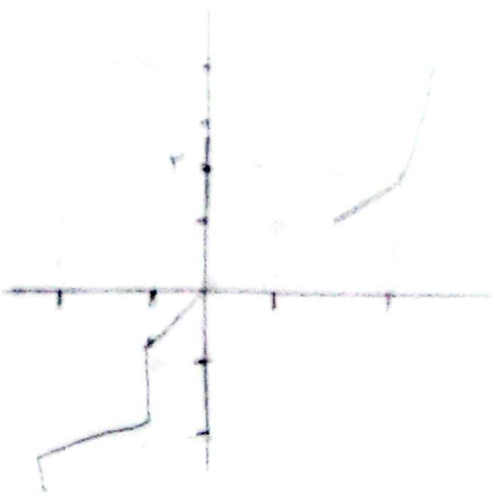
-۴ $[-2,2]$

الف) $y = [x] - 2x$

$$\begin{cases} -2 \le x < -1 \to -2x-2 \\ -1 \le x < 0 \to -2x-1 \\ 0 \le x < 1 \to -2x \\ 1 \le x < 2 \to -2x+1 \end{cases}$$



ب) $y = [x]|x|$



الف) $y = 3x - [3x] \longrightarrow R_f = [0,1)$

ب) $y = 4x - 4[x] \longrightarrow y = 4(x - [x]) \longrightarrow R_f = [0,4)$

ج) $y = x - 5\left[\frac{x}{5}\right] \longrightarrow y = 5\left(\frac{x}{5} - \left[\frac{x}{5}\right]\right) \longrightarrow R_f = [0,5)$

د) $y = [2x] - 2x \longrightarrow R_f = [-1,0)$

-6

الف) $y = 3\sin x + 2\cos x \longrightarrow y^2 = (3\sin x + 2\cos x)^2 = 9\sin^2 x + 12\sin x\cos x + 4\cos^2 x$

$y^2 = 4 + 5\sin^2 x + \underbrace{12\sin x\cos x}_{6\sin 2x} \longrightarrow -1 \le \sin x , \cos x \le +1 \longrightarrow [-\sqrt{13}, \sqrt{13}] = R_f$

ب) $y = 4\sin x - 3\cos x \longrightarrow -\sqrt{16+9} \le 4\sin x - 3\cos x \le \sqrt{16+9} \quad R_f = [-5,5]$

ج) $y = [2\sin x + 5\cos x]$

$-\sqrt{29} \le 2\sin x + 5\cos x \le \sqrt{29} \longrightarrow [2\sin x + 5\cos x] = \begin{cases} -6 \\ -5 \\ -4 \\ -3 \\ -2 \\ \vdots \\ 5 \end{cases}$

$R_f = \{-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

د) $\sqrt{\cos x - 2\sin x} \rightarrow \quad x - 2\sin x \ge 0 \rightarrow \cos x \ge 2\sin x \rightarrow \frac{\cos}{\sin} \ge 2$

$\ge 0 \rightarrow 0 \le f(x) \le \sqrt{\sqrt{5}} \rightarrow f(x) \in [0, \sqrt{5}] \qquad \cot x \ge 2$

$R_f = [0, \sqrt{5}]$

-7

الف) $\sin^2 x + 3\underbrace{(\sin x)}_{y} + 2 \qquad -1 \le y \le 1 \rightarrow y^2 + 3y + 2 \rightarrow y_{min} = \frac{-b}{2a} \rightarrow -1.5$

راس $\rightarrow (-1.5)^2 + 3(-1.5) + 2 = -0.25 \qquad y = -1 \rightarrow 1 - 3 + 2 = 0 , \; y = 1 \rightarrow 1 + 3 + 2 = 6$

$R_f = [0, 6]$

ب) $2\sin^2 x + 3\underbrace{(\sin x)}_{y} + 2 \longrightarrow -1 \le y \le 1 \rightarrow 2y^2 + 3y + 2 \rightarrow y_{min} = -\frac{b}{2a} = -0.75$

راس $\rightarrow 0.875 , \; y = -1 \rightarrow 1 , \; y = 1 \rightarrow 7 \rightarrow R_f = [0.875, 7]$

-۸

الف) $\sin^2 x + 4\cos^2 x \to (1-\cos^2 x) + 4\cos^2 x = 1+3\cos^2 x \to 0 \le \cos^2 x \le 1$

$\cos^2 x = 0 \to 1+3\cdot 0 = 1$ و $\cos^2 x = 1 \to 1+3\cdot 1 = 4 \to R_f = [1,4]$

ب) $\frac{2\cot x}{1+\cot^2 x} \to \frac{2\frac{\cos}{\sin}}{1+\frac{\cos^2}{\sin^2}} \to \frac{2\frac{\cos}{\sin}}{\frac{1}{\sin^2}} = \frac{2(\cos)(\cancel{\sin^2})}{\cancel{\sin}}$ $\sin$ $= 2\cos\sin$

$R_f = [-1,1]$

-۹

الف) $\sin^6 x + \cos^6 x \Rightarrow y_{min} = -\frac{b}{2a} \to \frac{1}{2} \to$ راس $= \frac{1}{4} \to (\sin^2 x + \cos^2 x)^3 = 1$

$\to R_f = [\frac{1}{4}, 1]$

ب) $[\sin^8 x + \cos^8 x] \to (\sin^2 x)^4 + (\cos^2 x)^4$

$\to 1-\sin^2 x \to \sin^2 x \in [0,1]$

$f(y) = 2y^4 - 4y^3 + 6y^2 - 4y + 1 \to f(0) = 1$, $f(1) = 1$ $R_f = \{0,1\}$

-۱۰

الف) $\frac{2x^2+7x-4}{x+4} \to \frac{2x^2+8x-x-4}{x+4} = \frac{(2x^2+8x)-(x+4)}{x+4}$

$f(x) = \frac{2x(x+4)-(x+4)}{x+4} \to \frac{(2x-1)(x+4)}{x+4} \to f(x) = 2x-1 \to$

$(-\infty,-9)\cup(-9,+\infty)$

ب) $\frac{x^3-1}{x-1} \to \frac{\cancel{(x-1)}(x^2+1+x)}{\cancel{x-1}} = x^2+x+1 = 0 \xrightarrow{x \neq 1}$ راس $x = \frac{-b}{2a} = -0/5$

راس $\to 0/75$ و $f(1) = 3 \to [0/75, +\infty)$