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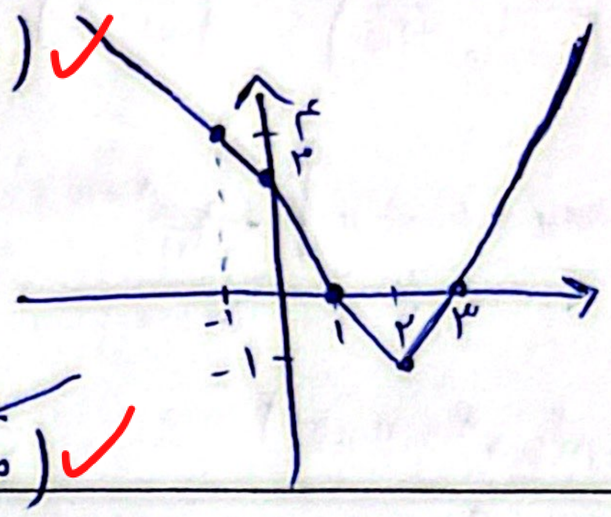
نام و نام خانوادگی ...آرمین میرزایی... پاسخنامه تشریحی تکلیف شماره ...۷... کلاس ...یازدهم ریاضی...

الف) $y = x + |2x-4|$

$2x-4 \geq 0 \Rightarrow x \geq 2$ ① $\quad 3x-4 \quad x \geq 2$

$-x+4 > 0 \Rightarrow x < 2$ ② $\quad -x+4 \quad x < 2$ (۲)

$1 \cup 2 \rightarrow y \geq 2 \Rightarrow R_f = [2, +\infty)$ ✓



$R_f = [-1, +\infty)$ ✓ ۱

ب) $y = |x-2| + |x-1| - |x|$

$f(2) = -1 \quad f(3) = 0$

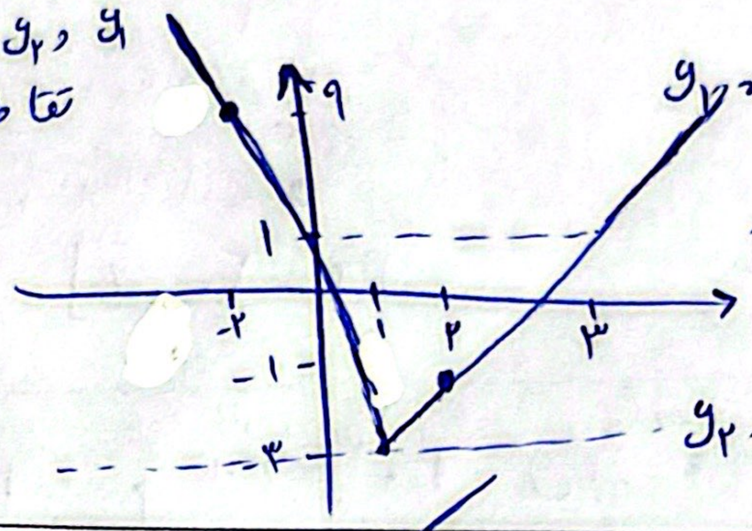
$f(1) = 0 \quad f(0) = 3 \quad f(-1) = 4$

$|3x-3| - |x+2| = k$

↓ یک ریشه دارد ⇒ g_1 و g_2 فقط در یک نقطه تقاطع داشتند.

$g_1 = |3x-3| - |x+2|$

$g_2 = k$



$g_2 = |3x-3| - |x+2|$

$f(-2) = 9 \quad f(2) = -1$

$f(0) = 1$

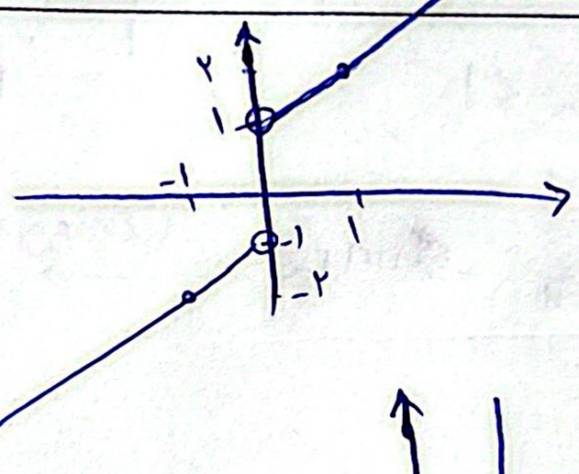
$f(1) = -3$

$g_2 = k \Rightarrow k = -3$ ✓ (۲) ۲

الف) $y = x + \frac{x}{|x|}$

$x+1 \quad x > 0$

$x-1 \quad x < 0$ (۲)

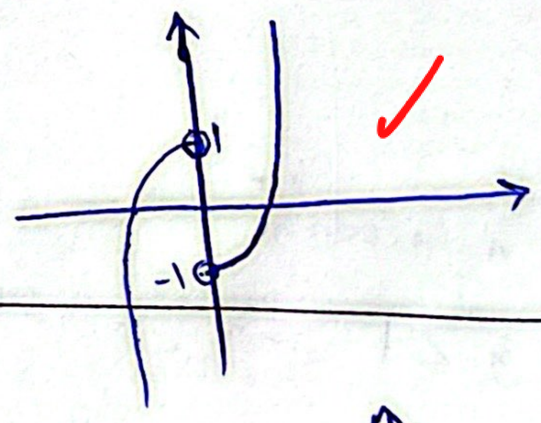


✓ ۳

ب) $y = x|x| - \frac{x}{|x|}$

$x^2 - 1 \quad x > 0$

$-x^2 + 1 \quad x < 0$



✓

الف) $y = [x] - 2x$

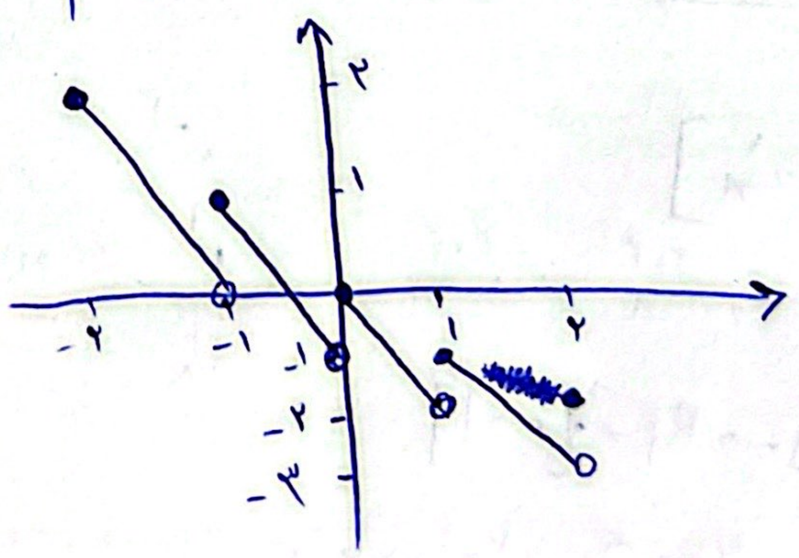
$-2 \leq x < -1 \rightarrow -2 - 2x$

$-1 \leq x < 0 \rightarrow -1 - 2x$

$0 \leq x < 1 \rightarrow -2x$

$1 \leq x < 2 \rightarrow 1 - 2x$

$x = 2 \rightarrow 2 - 2x \xrightarrow{x=2} y = -2$



✓ (۲) ۴

الف) $y = 3x - [3x]$

$a = 3x \rightarrow y = a - [a] \Rightarrow 0 \leq y < 1 \rightarrow R_f = [0, 1)$ ✓

ب) $y = 4x - 4[x] \Rightarrow y = 4(x - [x]) \xrightarrow{0 \leq x-[x]<1} 0 \leq 4(x-[x]) < 4$

$R_f = [0, 4)$ ✓

ج) $y = 5[\frac{x}{5}] = 5(\frac{x}{5} - [\frac{x}{5}]) \xrightarrow{0 \leq \frac{x}{5} - [\frac{x}{5}] < 1} 0 \leq 5(\frac{x}{5} - [\frac{x}{5}]) < 5$

$R_f = [0, 5)$ ✓

د) $y = [2x] - 2x \xrightarrow{a=2x} y = [a] - a \rightarrow -1 < [a] - a \leq 0 \rightarrow R_f = (-1, 0]$ ✓ (۲) ۵

$-\sqrt{a^2+b^2} \le a\sin x + b\cos x \le \sqrt{a^2+b^2}$

الف) $y = 3\sin x + 2\cos x$

$R_f = \left[-\sqrt{13}, \sqrt{13}\right]$ ✓

ب) $y = 2\sin x - 3\cos x$

$R_f = \left[-\sqrt{13}, \sqrt{13}\right] = [-a, a]$ ✓

ج) $y = [2\sin x + 3\cos x]$

$-\sqrt{13} \le 2\sin x + 3\cos x \le \sqrt{13}$

$-a \le a$

$R_f = \{-3, -2, \dots, 3\} = \{y \mid -3 \le y \le 3, y \in \mathbb{Z}\}$ ✓

د) $y = \sqrt{\cos x - 2\sin x}$

$\sqrt{0} \le \cos x - 2\sin x \le \sqrt{5}$

$\sqrt[4]{0} \le \sqrt[4]{\cos x - 2\sin x} \le \sqrt[4]{5}$

$R_f = [0, \sqrt[4]{5}]$ ✓

(۲)

الف) $y = \sin^2 x + 3\sin x + 2$

$= \left(\sin x + \frac{3}{2}\right)^2 - \frac{9}{4} + 2 = \left(\sin x + \frac{3}{2}\right)^2 - \frac{1}{4}$

$-1 \le \sin x \le 1$

$\frac{1}{2} \le \sin x + \frac{3}{2} \le \frac{5}{2}$

$\frac{1}{4} \le \left(\sin x + \frac{3}{2}\right)^2 \le \frac{25}{4}$

$0 \le \left(\sin x + \frac{3}{2}\right)^2 - \frac{1}{4} \le 6$

$R_f = [0, 6]$ ✓

(۲)

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الف) $y = \sin^2 x + 4\cos^2 x$ $\xrightarrow{\sin^2 x = 1 - \cos^2 x}$ $y = 1 + 3\cos^2 x$

$-1 \le \cos x \le 1 \rightarrow 0 \le \cos^2 x \le 1$

$1 \le y \le 4$

$R_f = [1, 4]$ ✓

(۲)

ب) $y = \frac{2\cot x}{1+\cot^2 x} = \frac{2\frac{\cos x}{\sin x}}{\frac{1}{\sin^2 x}} = 2\sin x \cos x = \sin 2\alpha$

$1 + \cot^2 x = \frac{1}{\sin^2 x}$

$-1 \le \sin\square \le 1 \rightarrow -1 \le \sin 2x \le 1$

$R_f = [-1, 1]$ ✓

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الف) $y = \sin^6 x + \cos^6 x = \sin^{3\times 2} x + \cos^{2\times 3} x$

$3^{1-k} \le \sin^{2k} x + \cos^{2k} x \le 1$ $\xrightarrow{-2}$ $3^{1-3} \le y \le 1$

$\rightarrow \frac{1}{4} \le y \le 1 \rightarrow R_f = \left[\frac{1}{4}, 1\right]$ ✓

(۲)

ب) $y = [\sin^8 x + \cos^8 x]$

$\sin^8 x + \cos^8 x = \sin^{2\times 4} x + \cos^{2\times 4} x$

$3^{2-4} \le \sin^8 x + \cos^8 x \le 1$

$\frac{1}{9} \le \sin^8 x + \cos^8 x \le 1$

$[\ ] \rightarrow [\sin^8 x + \cos^8 x] \rightarrow R_f = \{0, 1\}$ ✓

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١) $[x]\,|x|$

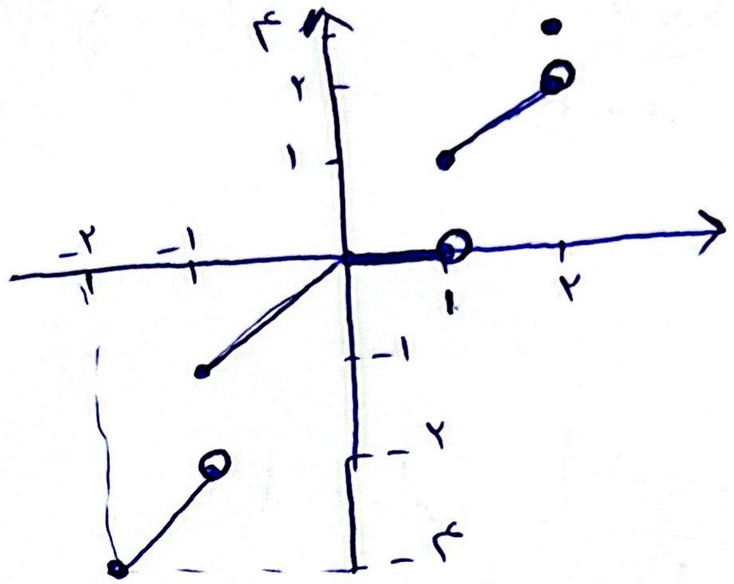
$-٢ \le x < -١ \rightarrow y = ٢x$

$-١ \le x < ٠ \rightarrow y = x$

$٠ \le x < ١ \rightarrow y = ٠$

$١ \le x < ٢ \rightarrow y = x$

$x = ٢ \rightarrow x = y$



✓

١) $y = ٢\sin^٢x + ٣\sin x + ٢ = A$

$\frac{١}{٢}A = \sin^٢x + \frac{٣}{٢}\sin x + ١ = \left(\sin x + \frac{٣}{٤}\right)^٢ - \frac{٩}{١٦} + ١ \quad \frac{٧}{١٦}$

$-\frac{١}{٤} \le \quad \le \frac{٧}{٤}$

$٠ \le \quad \le \frac{٤٩}{١٦}$

$\frac{٧}{١٦} \le \frac{١}{٢}A \le \frac{٤٩}{١٦} \; \frac{٢٨}{٨} \; \frac{١٤}{٤} \; \frac{٧}{٢}$

$\frac{٧}{٨} \le A \le ٧$

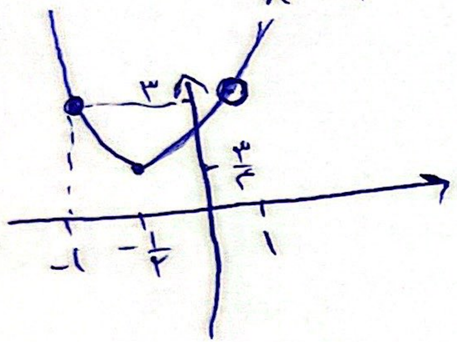
$R_f = \left[\frac{٧}{٨}, ٧\right]$ ✓

الف) $y = \frac{2x^2+7x-4}{x+4} = \frac{x^2+7x-8}{x+4} = \frac{(x+8)(2x-1)}{x+4} = 2x-1 \quad ; x \neq -4$ (۲)

$x \neq -4 \rightarrow y \neq -9$

$R_f = \mathbb{R} - \{-9\}$ ✓

ب) $y = \frac{x^3-1}{x-1} = \frac{(x-1)(x^2+x+1)}{x-1} = x^2+x+1$



$\rightarrow R_f = [\frac{3}{4}, +\infty)$ ✓