نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

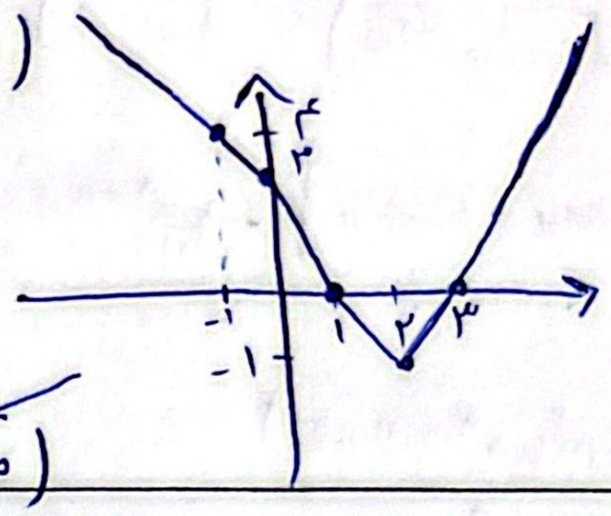
۱

الف) $y = x + |2x-4|$

$2x-4 \geq 0 \Rightarrow x \geq 2$ ① $\Leftarrow 3x-4 \quad x \geq 2$

$-x+4 > 0 \Rightarrow x < 2$ ② $\Leftarrow -x+4 \quad x < 2$

$1 \cup 2 \rightarrow y \geq 2 \Rightarrow R_f = [2, +\infty)$



ب) $y = |x-2| + |x-1| - |x|$

$f(2) = -1 \quad f(3) = 0$

$f(1) = 0 \quad f(0) = 3 \quad f(-1) = 4$

$R_f = [-1, +\infty)$

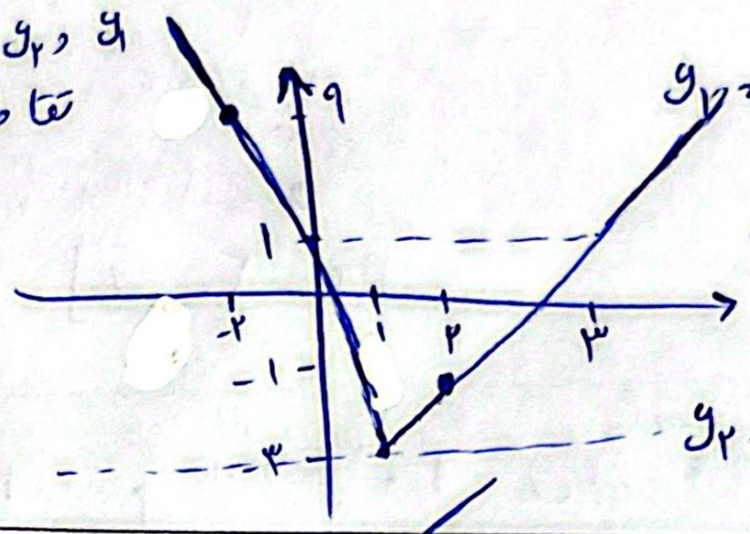
۲

$|3x-3| - |x+2| = k$

↓ یک ریشه دارد ⟹ g_1 و g_2 فقط در یک نقطه تقاطع داشته باشند.

$g_1 = |3x-3| - |x+2|$

$g_2 = k$



$g_2 = |3x-3| - |x+2|$

$f(-2) = 9 \quad f(2) = -1$

$f(0) = 1$

$f(1) = -3$

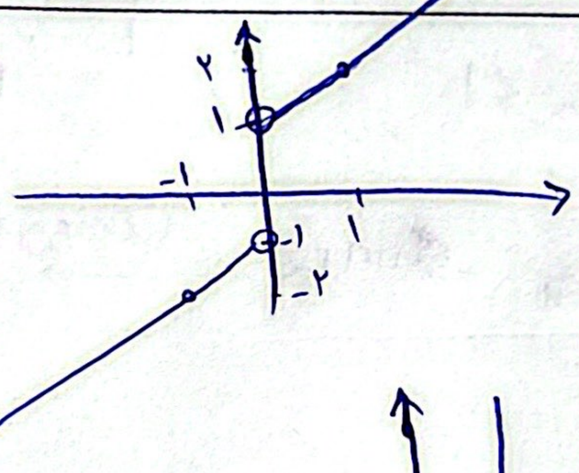
$g_2 = k \Rightarrow \underline{k = -3}$

۳

الف) $y = x + \frac{x}{|x|}$

$x > 0 \quad x + 1$

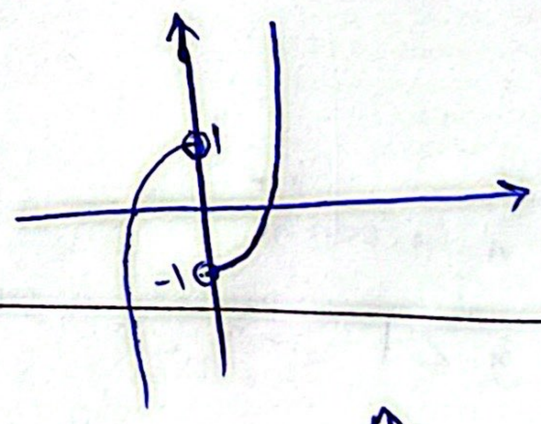
$x < 0 \quad x - 1$



ب) $y = x|x| - \frac{x}{|x|}$

$x > 0 \quad x^2 - 1$

$x < 0 \quad -x^2 + 1$



۴

الف) $y = [x] - 2x$

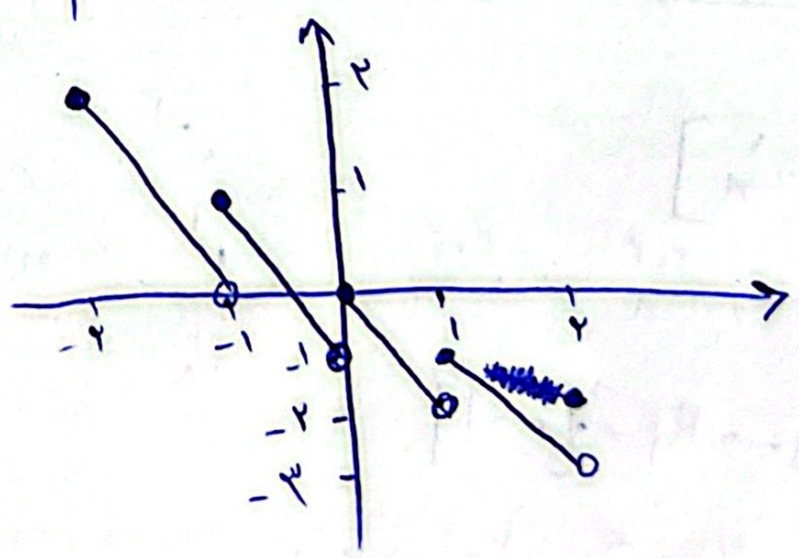
$-2 \leq x < -1 \rightarrow -2 - 2x$

$-1 \leq x < 0 \rightarrow -1 - 2x$

$0 \leq x < 1 \rightarrow -2x$

$1 \leq x < 2 \rightarrow 1 - 2x$

$x = 2 \rightarrow 2 - 2x \xrightarrow{x=2} y = -2$



۵

الف) $y = 3x - [3x]$

$a = 3x \rightarrow y = a - [a] \Rightarrow 0 \leq y < 1 \rightarrow R_f = [0, 1)$

ب) $y = 4x - 4[x] \Rightarrow y = 4(x - [x]) \xrightarrow{0 \leq x-[x]<1} 0 \leq 4(x-[x]) < 4$

$R_f = [0, 4)$

ج) $y = 5[\frac{x}{5}] = 5(\frac{x}{5} - [\frac{x}{5}]) \longrightarrow 0 \leq \frac{x}{5} - [\frac{x}{5}] < 1 \Rightarrow 0 \leq 5(\frac{x}{5} - [\frac{x}{5}]) < 5$

$R_f = [0, 5)$

د) $y = [2x] - 2x \xrightarrow{a=2x} y = [a] - a \rightarrow -1 < [a] - a \leq 0 \rightarrow R_f = (-1, 0]$

$-\sqrt{a^2+b^2} \leq a\sin x + b\cos x \leq \sqrt{a^2+b^2}$

الف) $y = 3\sin x + 4\cos x$

$R_f = \left[-\sqrt{13}, \sqrt{13}\right]$

ب) $y = 5\sin x - 3\cos x$

$R_f = \left[-\sqrt{34}, \sqrt{34}\right] = [-6, 6]$

ج) $y = \left[3\sin x + 6\cos x\right]$

$-6 \leq 3\sin x + 6\cos x \leq \sqrt{81}$

$-6 \leq 3\sin x + 6\cos x \leq 9$

$\left[3\sin x + 6\cos x\right] = \{-9, -8, \dots, 8\}$

$R_f = \{-9, -8, \dots, 8\} = \{y \mid -9 \leq y \leq 8, y \in \mathbb{Z}\}$

د) $y = \sqrt{\cos x - 2\sin x}$

$0 \leq \cos x - 2\sin x \leq 5$

$0 \leq \sqrt{\cos x - 2\sin x} \leq \sqrt{5}$

$R_f = [0, \sqrt{5}]$

الف) $y = \sin^2 x + 3\sin x + 2$

$= \left(\sin x + \frac{3}{2}\right)^2 - \frac{9}{4} + 2 = \left(\sin x + \frac{3}{2}\right)^2 - \frac{1}{4}$

$-1 \leq \sin x \leq 1$

$\frac{1}{2} \leq \sin x + \frac{3}{2} \leq \frac{5}{2}$

$\frac{1}{4} \leq \left(\sin x + \frac{3}{2}\right)^2 \leq \frac{25}{4}$

$0 \leq \left(\sin x + \frac{3}{2}\right)^2 - \frac{1}{4} \leq 6$

$R_f = [0, 6]$

الف) $y = \sin^2 x + 4\cos^2 x \xrightarrow{\sin^2 x = 1 - \cos^2 x} y = 1 + 3\cos^2 x$

$-1 \leq \cos x \leq 1 \rightarrow 0 \leq \cos^2 x \leq 1$

$1 \leq y \leq 4$

$R_f = [1, 4]$

ب) $y = \frac{2\cot x}{1+\cot^2 x} = \frac{2\frac{\cos x}{\sin x}}{\frac{1}{\sin^2 x}} = 2\sin x \cos x = \sin 2\alpha$

$-1 \leq \sin \square \leq 1 \quad -1 \leq \sin 2x \leq 1$

$R_f = [-1, 1]$

$1 + \cot^2 x = \frac{1}{\sin^2 x}$

الف) $y = \sin^6 x + \cos^6 x = \sin^{2\times 3} x + \cos^{2\times 3} x$

$1^{1-k} \leq \sin^{2k} x + \cos^{2k} x \leq 1$

$2^{1-3} \leq y \leq 1 \rightarrow \frac{1}{4} \leq y \leq 1 \rightarrow R_f = \left[\frac{1}{4}, 1\right]$

ب) $y = \left[\sin^8 x + \cos^8 x\right]$

$\sin^8 x + \cos^8 x = \sin^{2\times 4} x + \cos^{2\times 4} x$

$2^{1-4} \leq \sin^8 x + \cos^8 x \leq 1$

$\underbrace{2^{1-4}}_{\frac{1}{8}} \leq \sin^8 x + \cos^8 x \leq 1$

$[\,] \rightarrow \left[\sin^8 x + \cos^8 x\right] \rightarrow R_f = \{0, 1\}$

ـ) $[x]\,|x|$

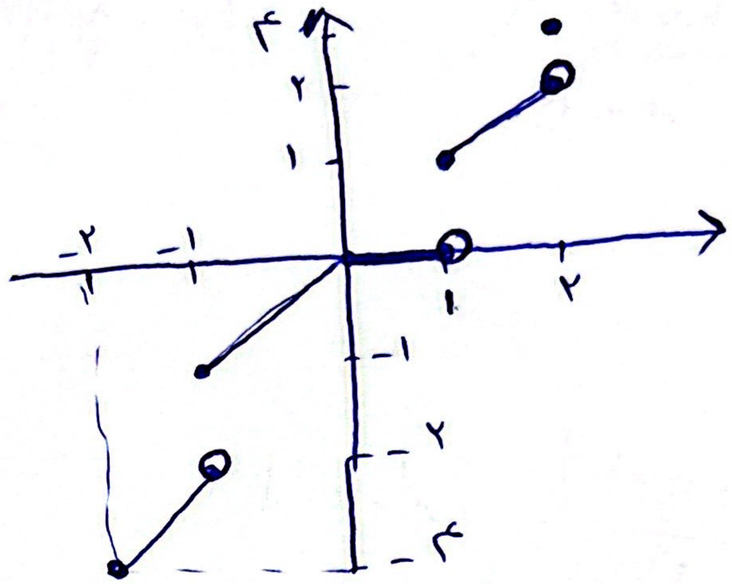
$-2 \le x < -1 \rightarrow y = 2x$

$-1 \le x < 0 \rightarrow y = x$

$0 \le x < 1 \rightarrow y = 0$

$1 \le x < 2 \rightarrow y = x$

$x = 2 \rightarrow x$



ـ) $y = 2\sin^2 x + 3\sin x + 2 = A$

$\frac{1}{2}A = \sin^2 x + \frac{3}{2}\sin x + 1 = \left(\sin x + \frac{3}{4}\right)^2 - \frac{9}{16} + 1 \quad \frac{7}{16}$

$-\frac{1}{4} \le \quad \le \frac{7}{4}$

$0 \le \quad \le \frac{49}{16}$

$\frac{7}{16} \le \frac{1}{2}A \le \frac{49}{16}\ \frac{28}{8}\ \frac{14}{4}\ \frac{7}{2}$

$\frac{7}{8} \le A \le 7$

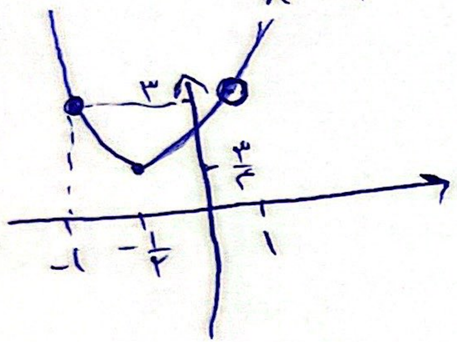
$R_f = \left[\frac{7}{8}, 7\right]$

الف) $y = \frac{2x^2+7x-6}{x+6} = \frac{x^2+7x-8}{x+6} = \frac{(x+8)(2x-1)}{x+6} = 2x-1 \quad ; x \neq -6$

$x \neq -6 \rightarrow y \neq -9$

$R_f = \mathbb{R} - \{-9\}$

ب) $y = \frac{x^3-1}{x-1} = \frac{(x-1)(x^2+x+1)}{x-1} = x^2+x+1$



$\rightarrow R_f = [\frac{3}{4}, +\infty)$