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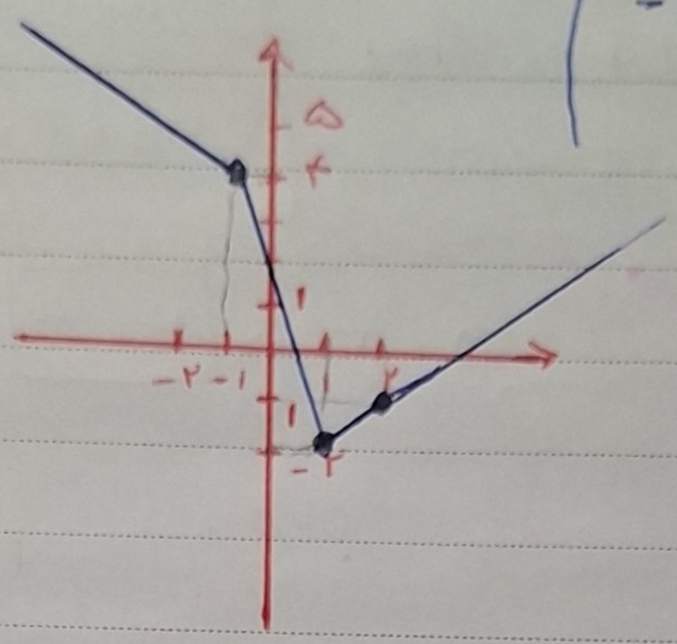
(10) ب) $y = |2x-2| - |x+1| \Rightarrow y = \begin{cases} 2x-2-x-1 & x>1 \\ -2x+2-x-1 & -1 \le x \le 1 \\ -2x+2+x+1 & x<-1 \end{cases}$

$\Rightarrow \begin{cases} x-3 & x>1 \\ -3x+1 & -1 \le x \le 1 \\ -x+3 & x<-1 \end{cases}$



$\Rightarrow R_f = [-2, +\infty)$

تکلیف ۷ امیرپویا رزاق زادگان - سیزدهم B

(7) الف) $y = x + |2x-4|$

$x \ge 2 \rightarrow y = x + 2x - 4 \Rightarrow y = 3x-4, x=2 \Rightarrow y=2 \Rightarrow y \in [2, +\infty)$ شرط ۱

$x < 2 \rightarrow y = x - 2x + 4 \Rightarrow y = -x+4, x=2 \Rightarrow y = -2+4 = 2 \Rightarrow y \in (2, +\infty)$ شرط ۲

$R_f = I \cup I \Rightarrow R_f = [2, +\infty)$

ب) $|x-2| + |x-1| - |x|$ شرط ۱ ← $x \ge 2$ ، شرط ۲ ← $1 \le x \le 2$ ، شرط ۳ ← $0 \le x \le 1$ ، شرط ۴ ← $x \le 0$

$x \ge 2 \rightarrow y = x-2+x-1-x = x-3, x=2 \Rightarrow y=-1 \Rightarrow y \in [-1, 0]$ ← شرط ۱

$1 \le x \le 2 \rightarrow y = 2-x+x-1-x = 1-x$ ($x=2 \Rightarrow y=-1$ ، $x=1 \Rightarrow y=0$) $\Rightarrow y \in [-1, 0]$ شرط ۲

$0 \le x \le 1 \rightarrow y = 2-x+1-x-x = 3-3x$ ($x=1 \Rightarrow y=0$ ، $x=0 \Rightarrow y=3$) $\Rightarrow y \in [3, +\infty)$ شرط ۳

$x \le 0 \rightarrow y = 2-x+1-x+x = 3-x, x=0 \Rightarrow y=3 \Rightarrow y \in [3, +\infty)$ شرط ۴

$R_f = ① \cup ② \cup ③ \cup ④ \Rightarrow R_f = [-1, +\infty)$

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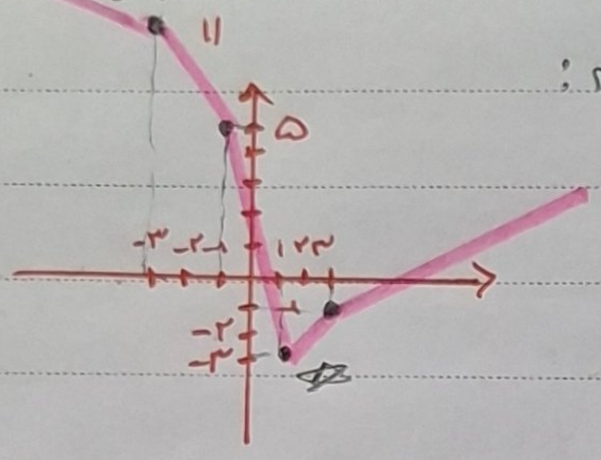
② $|۲x-۳|-|x+۲| \geq k$, k_{max} ؟

$f(x)=|۲x-۳|-|x+۲| \Rightarrow f(x)$ رسم می‌کنیم:

$$\begin{cases} ۲x-۳-x-۲ = x-۵ & x>۱ \\ -۲x+۳-x-۲ = -۴x+۱ & -۲\leq x\leq ۱ \\ -۲x+۳+x+۲ = -x+۵ & x<-۲ \end{cases}$$



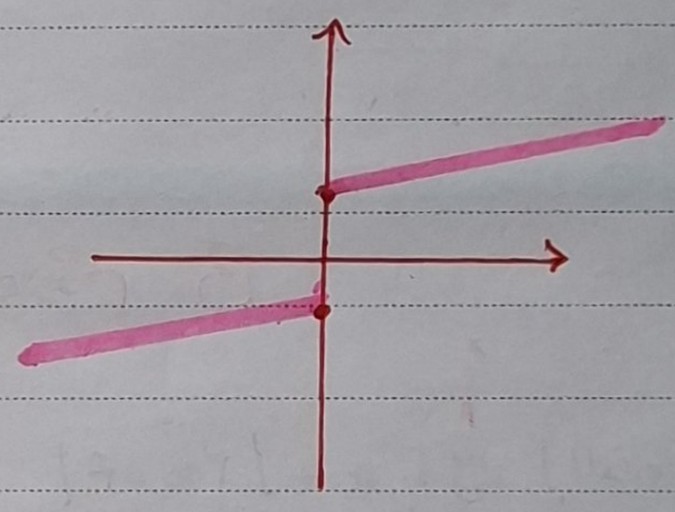
$y=-۳ \Rightarrow \boxed{k=-۳}$

③ الف)

$$y=x+\frac{x}{|x|}$$

$x>0$: $\frac{x}{|x|}=1 \Rightarrow y=x+1$

$x<0$: $\frac{x}{|x|}=-1 \Rightarrow y=x-1$

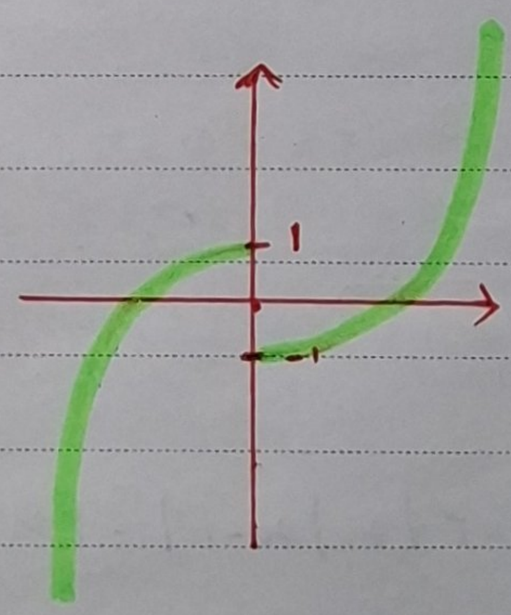


ب)

$$y=x|x|-\frac{x}{|x|}$$

$x>0$: $y=x^۲-1$

$x<0$: $y=-x^۲+1$

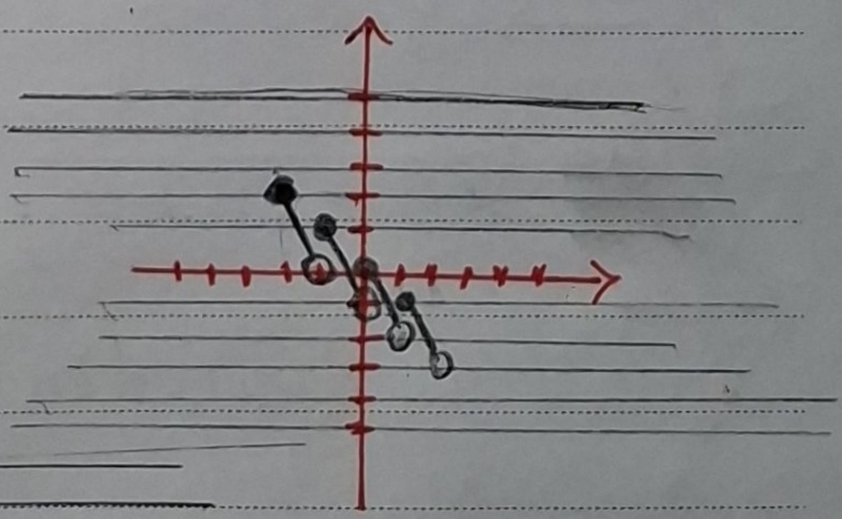


④ الف)

$$y=[x]-۲x$$

$x\in[-۲,-1) \Rightarrow -۲-۲x \Rightarrow x\in[-1,0) \Rightarrow -1-۲x$

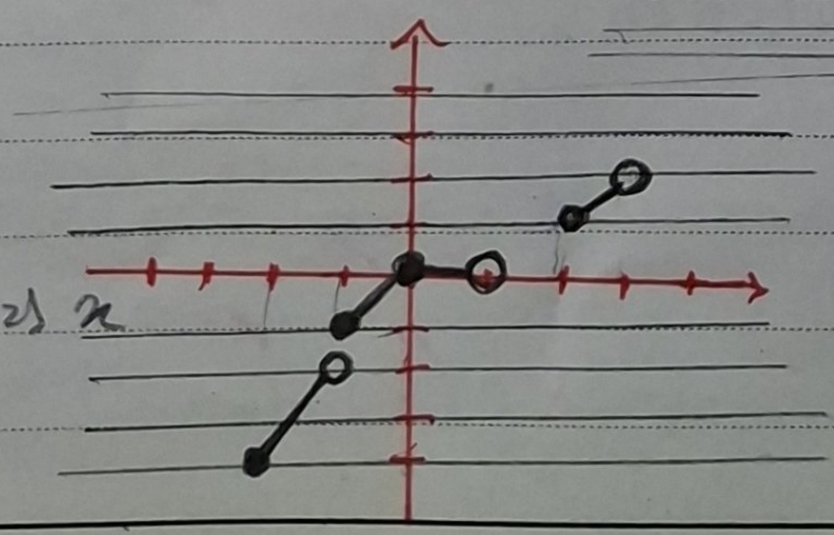
$x\in[0,1) \Rightarrow -۲x \Rightarrow x\in[1,۲) \Rightarrow 1-۲x$



ب) $y=[x]+|x|$

$x\in[-۲,-1) \Rightarrow (-۲)(-x)=۲x$

$x\in[0,1) \Rightarrow 0$, $x\in[1,۲) \Rightarrow x$



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(5) الف) $y = 3x - [3x] \Rightarrow y = t - [t] \Rightarrow R_f = [0,1)$

ب) $y = 4x - 4[x] \Rightarrow y = 4(x - [x]) \Rightarrow R_f = [0,4)$

ج) $y = x - 5\left[\frac{x}{5}\right] \Rightarrow y = 5t - 5[t] \Rightarrow y = 5(t - [t]) \Rightarrow R_f = [0,5)$

د) $y = [2x] - 2x \Rightarrow y = [t] - t \Rightarrow R_f = [-1,0)$

(6) الف) $y = 3\sin(x) + 2\cos(x)$

$\Rightarrow \sqrt{-9+4} \leq 3\sin x + 2\cos x \leq \sqrt{9+4} \Rightarrow R_f = [-\sqrt{13}, \sqrt{13}]$

ب) $y = 4\sin(x) - 3\cos(x)$

$\Rightarrow -\sqrt{16+9} \leq 4\sin x - 3\cos x \leq \sqrt{16+9} \Rightarrow R_f = [-5, 5]$

ج) $y = [2\sin(x) + 5\cos x]$

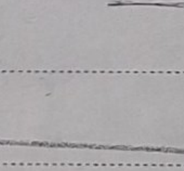
$-\sqrt{29} \leq \sin x + 5\cos x \leq \sqrt{29} \Rightarrow R_f = [2\sin x + 5\cos x]$

$= \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

د) $y = \sqrt{\cos x - 2\sin x} \Rightarrow -\sqrt{5} \leq \cos x - 2\sin x \leq \sqrt{5}$

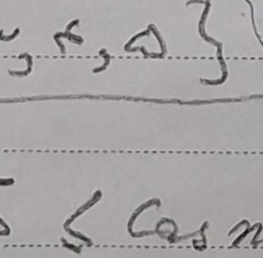
$\Rightarrow 0 \leq \sqrt{\cos(x) - 2\sin x} \leq \sqrt{5} \Rightarrow R_f = [0, \sqrt{5}]$

(7) الف) $y = \sin^2 x + 2\sin x + 2$



$\Rightarrow R_f = [0, 6]$

ب) $y = 2\sin^2 x + 3\sin x + 2$



$\Rightarrow R_f = [\frac{1}{8}, 7]$

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(8) الف)

$y = \sin^2 x + 4\cos^2 x \Rightarrow y = \underbrace{\sin^2 x + \cos^2 x}_{(1)} + 3\cos^2 x$

$\Rightarrow y = 1 + 3\cos^2(x)$, $0 \leq \cos^2 x \leq 1 \Rightarrow 0 \leq 3\cos^2 x \leq 3 \Rightarrow$

$1 \leq 1 + 3\cos^2 x \leq 4 \Rightarrow \boxed{R_f = [1, 4]}$

ب) $y = \dfrac{2\cot x}{1 + \cot^2 x} \Rightarrow y = 2 \times \dfrac{\cos x}{\sin x} \times \sin^2 x = 2\sin x \cdot \cos x$

$= \sin 2x \Rightarrow y = \sin 2x \Rightarrow \boxed{R_f = [-1, 1]}$

(9) $y = \sin^4 x + \cos^4 x \Rightarrow \frac{1}{4} \leq \sin^4 x + \cos^4 x \leq 1$

$\Rightarrow R_f = [\frac{1}{4}, 1]$

ب) $y = [\sin^8 x + \cos^8 x] \Rightarrow \frac{1}{1} \leq \sin^8 x + \cos^8 x \leq 1$

$\Rightarrow [\sin^8 x + \cos^8 x] = \begin{cases} 0 \\ 1 \end{cases} \Rightarrow \boxed{R_f = \{0, 1\}}$

(10)

الف) $y = \dfrac{2x^2 + 7x - 4}{x + 4} \Rightarrow y = \dfrac{(x+4)(2x-1)}{x+4} \Rightarrow y = (2x-1), (x \neq \underbrace{-4}_{\swarrow})$

$2(-4) - 1 = -9 \Rightarrow \boxed{R_f = \mathbb{R} - \{-9\}}$

ب) $y = \dfrac{x^3 - 1}{x - 1} \Rightarrow y = \dfrac{(x-1)(x^2 + x + 1)}{x-1} \Rightarrow y = x^2 + x + 1$

$\Rightarrow y = (x + \frac{1}{2})^2 + \frac{3}{4} \Rightarrow R_f = [y_{min}, +\infty) \Rightarrow \boxed{R_f = [\frac{3}{4}, +\infty)}$

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