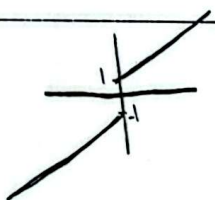


الف) $x + |2x - 4| \geq |2x - 4| \rightarrow \begin{cases} 2x - 4 & x \geq 2 \xrightarrow{+x} 3x - 4 \\ 4 - 2x & x < 2 \xrightarrow{+x} 4 - x \end{cases}$
 $\rightarrow$ برد: $[2, +\infty)$

ب) $|x - 2| + |x - 1| - |x| \rightarrow \begin{cases} x < 0 \rightarrow 3 - x \rightarrow (3, +\infty) \\ 0 \leq x < 1 \rightarrow 3 - 2x \rightarrow [0, 3] \\ 1 \leq x < 2 \rightarrow 1 - x \rightarrow (-1, 0] \\ x \geq 2 \rightarrow x - 2 \rightarrow [-1, +\infty) \end{cases}$
 $\rightarrow$ برد $\rightarrow [-1, +\infty)$

$|3x - 3| - |x + 2| = k$
 $x < -2 \rightarrow -2x + 3 + x + 2 \geq -2x + 5 \rightarrow (9, +\infty)$
 $-2 \leq x < 1 \rightarrow -2x + 3 - x - 2 \geq -3x + 1 \rightarrow [-3, 9]$
 $x \geq 1 \rightarrow 2x - 3 - x - 2 \geq x - 5 \rightarrow [-3, +\infty)$
 $\rightarrow k \geq -3$

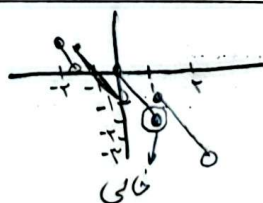
الف) $x + \frac{x}{|x|}$



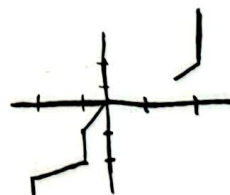
ب) $y = x|x| - \frac{x}{|x|}$



الف) $[x] - 2x$



ب) $[x]|x|$



الف) $2x - [2x] \rightarrow x - [x] \rightarrow 0 \leq x - [x] < 1 \rightarrow$ برد: $[0, 1)$

ب) $x - [x] \rightarrow x - [x] \rightarrow 0 \leq x - [x] < 1 \rightarrow$ برد: $[0, 1)$

ج) $x - 5[\frac{x}{5}] \rightarrow 5x \leq x < 5x + 5 \rightarrow f(x) = x - 5k \rightarrow 0 \leq x < 5 \rightarrow$ برد: $[0, 5)$

د) $[2x] - 2x \rightarrow 0 \leq y < 1 \rightarrow 0 \leq 2x < 1 \rightarrow -(2x) \rightarrow -1 < f(x) \leq 0 \rightarrow$ برد: $[-1, 0)$

y

1) $r \sin \alpha + r \cos \alpha \rightarrow (r \sin \alpha + r \cos \alpha)^2 = 9 \sin^2 \alpha + r^2 \cos^2 \alpha + 18 \sin \alpha \cos \alpha$
 $\rightarrow y^2 = r^2 + 18 \sin \alpha \cos \alpha \rightarrow -1 \leq \sin \alpha, \cos \alpha \leq 1 \rightarrow [-\sqrt{19}, \sqrt{19}]$
 2) $r \sin \alpha - r \cos \alpha = R \sin(\alpha + \alpha) \rightarrow R = \sqrt{50} \rightarrow f(x) = \Delta \sin(\alpha + \alpha)$
 $\rightarrow x \in [-\Delta, \Delta]$
 3) $[r \sin \alpha + \Delta \cos \alpha] \rightarrow R = \sqrt{29} \rightarrow g(x) \in [-\sqrt{29}, \sqrt{29}] \rightarrow x \in \{-\Delta, -f, \dots, \Delta\}$
 4) $\sqrt{r \cos \alpha - r \sin \alpha} \rightarrow \sin \alpha \rightarrow 0 \leq f(x) \leq \sqrt{20} \rightarrow f(x) \in [0, \sqrt{20}]$

الف) $\sin^2 \theta + 2 \sin \theta + 2 \rightarrow -1 \leq y \leq 1 \rightarrow y^2, 2y + 2 \rightarrow y_{\min} = -1, 8$
 في $y \rightarrow (-1, 8) + 2(-1, 8) + 2 = 2, 18 - 4, 8 + 2 = 0, 18$
 $y_2 = 1 \rightarrow 1 - 3 + 2 = 0, y_2 = 1 \rightarrow 1 + 2 + 2 = 5 \rightarrow y: [0, 5]$

ب) $2 \sin^2 \theta + 2 \sin \theta + 2 \rightarrow -1 \leq y \leq 1 \rightarrow 2y^2 + 2y + 2 \rightarrow y_{\min} = -0, 178$
 في $y \rightarrow 0, 178, y = -1 \rightarrow 1, y_2 = 1 \rightarrow 5 \rightarrow [0, 178, 5]$

۱) $\sin^2 \theta + \cos^2 \theta \rightarrow (1 - \cos^2 \theta) + \cos^2 \theta = 1 + \cos^2 \theta \rightarrow 0 \leq \cos^2 \theta \leq 1$
 $\cos^2 \theta \geq 0 \rightarrow 1, \cos^2 \theta \leq 1 \rightarrow \cos \theta \rightarrow$ برد: $[1, 1]$

۲) $\frac{\cos \theta}{1 + \cos^2 \theta} \rightarrow \frac{\cos \theta}{\sin^2 \theta} \rightarrow \left\{ \frac{\cos \theta}{\sin^2 \theta} \right\} = \frac{\cos \theta (\sin^2 \theta)}{\sin^2 \theta} \rightarrow \cos \theta \sin \theta$
 $\frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta} \rightarrow$ برد: $[-1, 1]$

$$\begin{aligned} \text{الف) } \sin^2 x + \cos^2 x = y_{\min} = \frac{-b}{2a} &\rightarrow \frac{1}{4} \rightarrow \text{و } \frac{1}{4} \rightarrow \sin^2 x + \cos^2 x = 1 \\ &\hookrightarrow \text{من: } \left[\frac{1}{4}, 1\right] \\ \text{ب) } [\sin^4 x + \cos^4 x] &\rightarrow (\sin^2 x)^2 + (\cos^2 x)^2 \\ &\hookrightarrow 1 - \sin^2 x \rightarrow \sin^2 x \in [0, 1] \\ f(y) = 2y^4 - 4y^3 + 4y^2 - 4y + 1 &\rightarrow f(0) = 1 \\ &\quad f(1) = 1 \rightarrow \text{من: } \{0, 1\} \end{aligned}$$

الف) $\frac{x^5 + \sqrt{x} - t}{x+t} \rightarrow \frac{x^5 + \sqrt{x} - x - t}{x+t} = \frac{(x^5 + \sqrt{x}) - (x+t)}{x+t}$
 $f(x) = \frac{x(x+t) - (x+t)}{x+t} \rightarrow \frac{(x-1)(x+t)}{x+t} \xrightarrow{x \neq -t} f(x) = x-1$
 ب) $\frac{x^5 - 1}{x-1} \rightarrow \frac{(x-1)(x^4 + x^3 + x^2 + x + 1)}{x-1} = x^4 + x^3 + x^2 + x + 1 \xrightarrow{x \neq 1} \lim_{x \rightarrow 1} x = 5$
 ج) $\lim_{x \rightarrow 1} f(x) = 5 \rightarrow [0, 5], +\infty)$