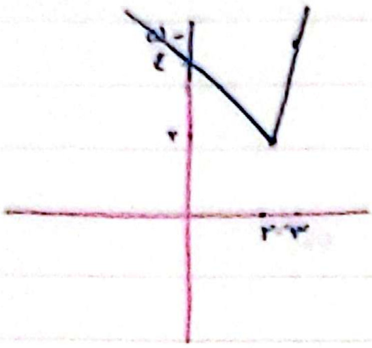
سپهر علایی

Date No

۱- الف)

$y = x + |2x - 4|$

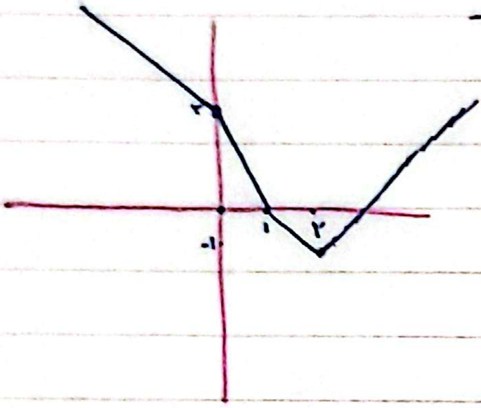


	2	
$4 - x$	2	$3x - 4$

$R_f = [2, +\infty)$

ب)

$y = |x - 2| + |x - 1| - |x|$

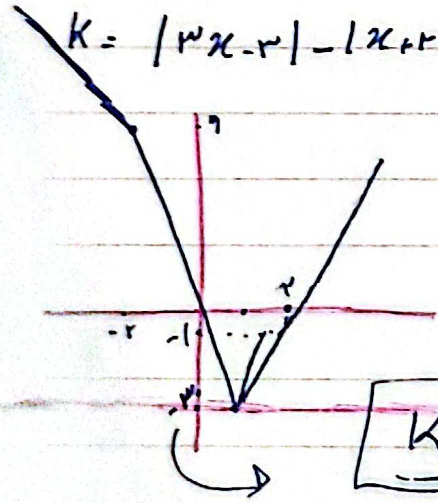


	0		1		2	
$3 - x$	3	$3 - 3x$	0	$1 - x$	-1	$x - 3$

$R_f = [-1, +\infty)$

۲-

$K = |3x - 3| - |x + 2|$



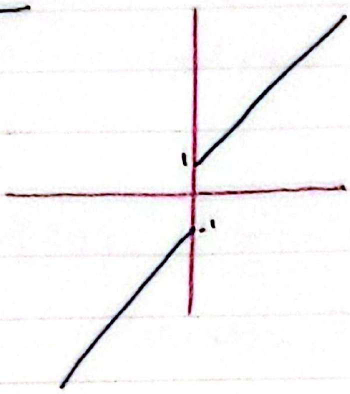
	-2		1	
$5 - 2x$	9	$1 - 4x$	-3	$2x - 5$

$\boxed{K = -3}$

٣ - الف)

$y = x + \frac{x}{|x|}$

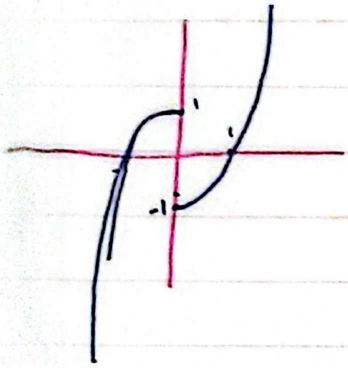
$$\begin{array}{c|c} x-1 & x+1 \end{array} \quad (0)$$



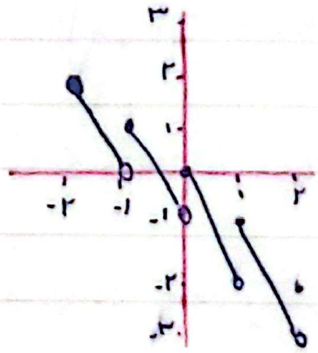
ب)

$y = x|x| - \frac{x}{|x|}$

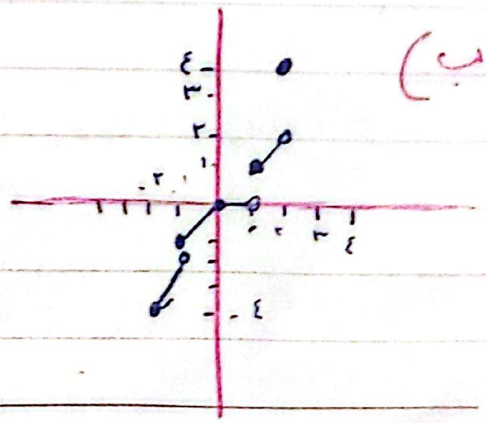
$$\begin{array}{c|c} -x^2+1 & x^2-1 \end{array} \quad (0)$$



٤ - الف)



ب)



Date No

۵

الف) $y = 11 \times 4 x 6 2$

$0 \le \bigcirc - [\bigcirc] < 1$

$0 \le ۳x - [۳x] < 1$

$R_f = [0, 1)$

ب) $y = ۴x - ۴[x] = ۴(x - [x]) \Rightarrow R_f = [0, ۴)$

ج) $(۵ \times \frac{x}{۵}) - ۵[\frac{x}{۵}] =$

$R_f = [0, ۵)$

$۵(\frac{x}{۵} - [\frac{x}{۵}]) \Rightarrow R_f = [0, ۵)$

د) $[۲x] - ۲x$ $\qquad -1 < ۲x - [۲x] \le 0$

$R_f = (-1, 0]$

۶_ الف)

$-\sqrt{a^۲ + b^۲} \le a\sin x + b\cos x \le \sqrt{a^۲ + b^۲}$

$-\sqrt{۹ + ۴} \le ۳\sin x + ۲\cos x \le \sqrt{a^۲ + b^۲} \Rightarrow R_f = [-\sqrt{۱۳}, \sqrt{۱۳}]$

ب) $-\sqrt{۲۵} \le ۴\sin x - ۳\cos x \le \sqrt{۲۵} \Rightarrow R_f = [-۵, ۵]$

ج) $-\sqrt{۲۹} \le ۲\sin x + ۵\cos x \le \sqrt{۲۹} \Rightarrow$ ~~$R_f = \{\dots\}$~~

$R_f = \{-۶, -۵, -۴, -۳, -۲, -۱, 0, ۱, ۲, ۳, ۴, ۵\}$

د) $-\sqrt{۵} \le \cos x - ۲\sin x \le \sqrt{۵} \Rightarrow R_f =$

dotnote

$R_f = [0, \sqrt[4]{۵}]$

۷- الف)

$y = \sin^2 x + 3\sin x + 2$

$\sin x = 0 \Rightarrow y = 2$

$\sin x = 1 \Rightarrow y = 6$

$\frac{-b}{2a} = -\frac{3}{2} \Rightarrow \frac{9}{4} - \frac{9}{2} + 2 = -\frac{1}{4}$

زیرا بازه $\sin x$ بین $[-1, 1]$ است → غ ق ق

$R_f = [0, 6]$

ب)

$y = 2\sin^2 x + 3\sin x + 2$

$\sin x = -1 \Rightarrow y = 1$

$\sin x = 1 \Rightarrow y = 7$

$\frac{-b}{2a} = \frac{-3}{4} \Rightarrow \frac{9}{8} + -\frac{9}{4} + 2$

$-\frac{9}{8} + 2 = \frac{7}{8}$

$R_f = [\frac{7}{8}, 7]$

۸- الف)

$y = \sin^2 + 4\cos^2 = \sin^2 + \cos^2 + 3\cos^2 = 3\cos^2 + 1$

$y = 1 + 3\cos^2$

$R_f = [1, 4]$

$y_{min} = 1 \quad y_{max} = 4$

ب)

$y = \frac{2\cot x}{1 + \cot^2 x} = \frac{2\cot x}{\frac{1}{\sin^2}} = \frac{\frac{2\cos x}{\sin x}}{\frac{1}{\sin^2 x}} = 2\sin x \times \cos x =$

$y = \frac{\sin 2x}{2} \Rightarrow R_f = [-1, 1]$

$$۲^{1-۳} \leq \sin^{۶} + \cos^{۶} \leq 1 \Rightarrow R_f = [\frac{1}{۴}, 1]$$

ب) $$\left[\frac{1}{۸} \leq \sin^{۸} + \cos^{۸} \leq 1\right] \Rightarrow \begin{cases} 0 \\ 1 \end{cases} \quad \boxed{R_f = \{0, 1\}}$$

۱۰ - الف) $$y = \frac{(x+۴)(۲x-1)}{(x+۴)} = ۲x-1 \quad \boxed{R_f = R - \{-۹\}}$$

$$x \neq -۴ \Rightarrow y \neq -۸-1 = -۹$$

ب) $$y = \frac{(x-1)(x^{۲}+x+1)}{x-1} = x^{۲}+x+1$$

$$x = \frac{-b}{۲a} = \frac{-1}{۲} \Rightarrow y_{min} = \frac{1}{۴} - \frac{1}{۲} + 1 = \frac{۳}{۴}$$

$$\boxed{R_f = [\frac{۳}{۴}, +\infty)}$$