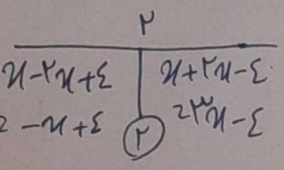
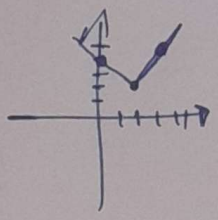
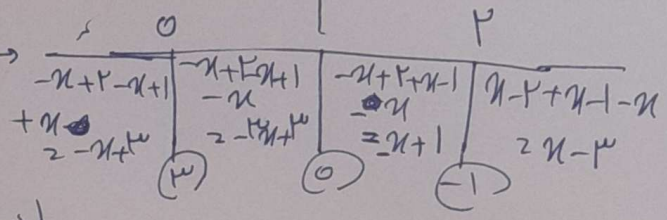
1) الف) $y = x + |x^2 - 3|$





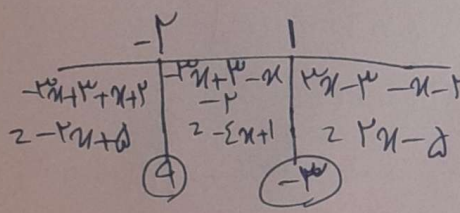
$R_y = [2, +\infty)$

ب) $y = |x^2 - 2x| + |x - 1| - |x|$



$\rightarrow R_y = [-1, +\infty)$

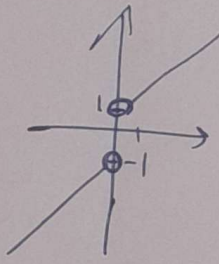
2) $y = |x^2 - 3x| - |x + 2|$



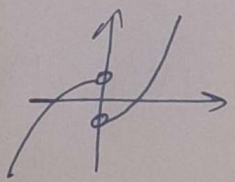
برای اینکه یک ریشه داشته باشد

$\rightarrow x = 3$

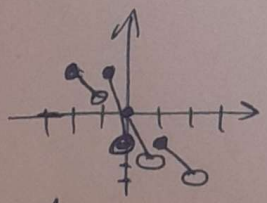
3) الف) $y = x + \frac{x}{|x|} \rightarrow y = \begin{cases} x > 0 & x + 1 \\ x < 0 & x - 1 \\ x = 0 & \text{ت ن} \end{cases}$



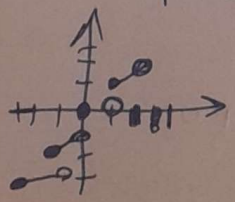
ب) $y = x|x| - \frac{|x|}{x} \rightarrow y = \begin{cases} x > 0 & x^2 - 1 \\ x < 0 & -x^2 + 1 \\ x = 0 & \text{ت ن} \end{cases}$



4) الف) $y = x - [x]$



ب) $y = [x]|x|$



5) الف) $y = x^3 - [x^3] \rightarrow 0 \le x^3 - [x^3] < 1 \rightarrow R_y = [0, 1)$

ب) $y = 3x - 3[x] \rightarrow 3(x - [x]) \rightarrow R_y = [0, 3)$

$[0, 1)$

ج) $y = x - 5[\frac{x}{5}] \rightarrow 5(\frac{x}{5} - [\frac{x}{5}]) \rightarrow R_y = [0, 5)$

$[0, 1)$

ب) $y = [\sin x] - \sin x \rightarrow -1 \le [\sin x] - \sin x < 0 \rightarrow R_y = [-1, 0)$

۶) الف) $y = 3\sin x + 2\cos x \rightarrow -\sqrt{9+4} \le \underbrace{3\sin x + 2\cos x}_{A} \le \sqrt{9+4}$

$\rightarrow -\sqrt{13} \le A \le \sqrt{13} \rightarrow R_y = [-\sqrt{13}, \sqrt{13}]$

ب) $y = 5\sin x - 12\cos x \rightarrow -\sqrt{144+25} \le \underbrace{5\sin x - 12\cos x}_{B} \le \sqrt{144+25}$

$\rightarrow -\sqrt{169} \le B \le \sqrt{169} \rightarrow R_y = [-13, 13]$

ج) $y = [3\sin x + 4\cos x] \rightarrow -\sqrt{25} \le [3\sin x + 4\cos x] \le \sqrt{25}$

$\rightarrow -5 \le [3\sin x + 4\cos x] \le 5$

$\rightarrow R_y = [-5, 5]$

د) $y = \sqrt{\cos x - 2\sin x} \rightarrow -\sqrt{5} \le 2\cos x - 2\sin x \le 5$

$\rightarrow 0 \le 2\cos x - 2\sin x \le \sqrt{5} \rightarrow R_y = [0, \sqrt{5}]$

۷) الف) $y = \sin^2 x + 3\sin x + 2$

$\sin x$: $1 \rightarrow 6$, $-1 \rightarrow 0$, $-\frac{1}{2}$ غ ق ق

$-\frac{b}{2a} = -\frac{3}{2}$

$\Rightarrow R_y = [0, 6]$

ب) $y = 2\sin^2 x + 3\sin x + 2 \rightarrow \sin x$: $1 \rightarrow 5$, $-1 \rightarrow 1$, $\frac{5}{8}$

$-\frac{b}{2a} = -\frac{3}{4}$

$\rightarrow R_y = [\frac{1}{8}, 5]$

۸) الف) $y = 3\sin^2 x + 4\cos^2 x \rightarrow y = 3\sin^2 x + 4(1 - \sin^2 x) = 3\sin^2 x + 4 - 4\sin^2 x$

$= -\sin^2 x + 4 \rightarrow R_y = [0, 4]$

$\sin^2 x \in [0, 1]$

$-\sin^2 x \in [-1, 0]$

ب) $y = \frac{2\cot x}{1 + \cot^2 x} = \frac{2\frac{\cos x}{\sin x}}{\frac{1}{\sin^2 x}} = 2\sin x \cos x = \sin 2x \rightarrow R_y = [-1, 1]$

۹) الف) $y = \sin^6 x + \cos^6 x$

$2k = 6 \rightarrow k = 3$

$\rightarrow 2^{-2} \le \underbrace{\sin^6 x + \cos^6 x}_{A} \le 1 \rightarrow \frac{1}{4} \le A \le 1$

$\rightarrow R_y = [\frac{1}{4}, 1]$

ب) $y = [5\sin^4 x + \cos^4 x]$

$2k = 4 \rightarrow k = 2$

$-2 \le \sin^4 x + \cos^4 x \le 1$

$\rightarrow \frac{1}{4} \le \underbrace{\sin^4 x + \cos^4 x}_{B} \le 1$ ← ماکزیمم ← $\rightarrow 0 \le B \le 1$ ← مینیمم

$\rightarrow R_y = [0, 1]$

٥) ب) $y = \frac{2x^2+5x-3}{x+3} \rightarrow 2x^2+5x-3 \;\big|\; x+3$, $2x-1$

$-2x^2-6x$

$-x-3$

$+x+3$

0

$\rightarrow y = \frac{(x+3)(2x-1)}{(x+3)} = 2x-1 \rightarrow x \neq -3 \rightarrow y \neq -7$

$\rightarrow R_y = R - \{-7\}$

ج) $y = \frac{1-x^3}{1-x} = \frac{(1-x)(1+x+x^2)}{(1-x)} = 1+x+x^2$

$\rightarrow \frac{-b}{2a} = \frac{-1}{2} \rightarrow +\frac{1}{3} - \frac{2}{3} + 1 = \frac{3}{3} = y_{min}$

$\rightsquigarrow R_y = [\frac{3}{4}, +\infty) - \{2\} \rightarrow x = 1 \rightarrow y = 3$