

$$1-x^2 \geq 0 \Rightarrow D_f = [-1, 1]$$

$$g.f = \{(-1, 2), (0, 4), (1, 4)\}$$

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$$f(x) = 2x - 1 \quad P_f = [3, +\infty) \Rightarrow R_f = [4, +\infty) (I)$$

$$g(x) = \frac{1}{x} x + 3 \quad D_g = (-\infty, 3] \Rightarrow R_g = (-\infty, 4] (II)$$

$$\Rightarrow I \cup II = (-\infty, 4] \cup [4, +\infty)$$

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$$-\frac{x^2}{x} + x + 3 > \frac{3}{x} \rightarrow -x^2 + 2x + 9 > 3 \rightarrow -x^2 + 2x + 6 > 0 \rightarrow$$

$$-(x-3)(x+2) > 0 \quad \frac{-1}{-} + \frac{3}{-} \Rightarrow (a, b) = (-1, 3)$$

$$\Rightarrow \sqrt{3 - (-1)} = \sqrt{4} = 2$$

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$-4x+1$	$-2x+6$	$2x-2$
$(-\infty, +\infty)$	$(2, 4)$	$(4, +\infty)$

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(2, 4)

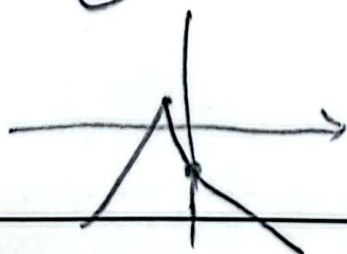
$$\min R_f = 2$$

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-1	0
$x+2$	$-2x-2$
(1)	(-2)

$$R_f = (-\infty, 1]$$

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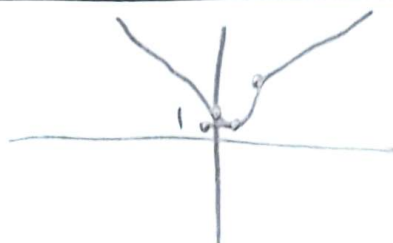
$$y^r + y = x^r + a x + m \Rightarrow x^r + (a-y)x + (m-y) = 0$$

$$\Rightarrow \Delta \geq 0 \rightarrow (a-y)^r - 4(m-y) \geq 0 \rightarrow y^r - r y + r a - r m$$

$$\xrightarrow{-\frac{b}{r a} = r} a - 1 + r a - r m \Rightarrow r m \geq 1 \Rightarrow m \leq r$$

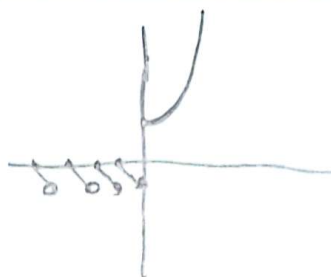
$$\xrightarrow{m \neq \frac{1}{r} \text{ and } m \in \mathbb{Z}} m = \{1, 2, r\}$$

$$f(x) = \begin{cases} x+r: x \geq r \\ x^r - r x + r: 0 \leq x < r \\ |x|+r: x < 0 \end{cases}$$



$$R_f = [1, +\infty)$$

$$f(x) = \begin{cases} x^r + r x + r: x \leq 0 \\ [x] - r x: x > 0 \end{cases}$$



$$R_f = [-1, 0] \cup [r, +\infty)$$

$$y = a + 1 - \sqrt{r x + r} \quad D_f = [b, +\infty) \rightarrow r x + r \geq 0 \rightarrow x \geq -\frac{r}{r}$$

$$R_f = (-\infty, a] \rightarrow \underbrace{-\sqrt{r x + r} + a + 1}_{\max x \leq 0} = a$$

$$\Rightarrow b \leq -\frac{r}{r}$$

$$\Rightarrow a + 1 = a \Rightarrow a = r$$

$$a b = f\left(x = -\frac{r}{r}\right) = -r$$

$$f(x) + g(x) = r \sqrt{(\sqrt{x+1} + \sqrt{1-x})^r} \rightarrow f(x) + g(x) = r \sqrt{x+1} + r \sqrt{1-x}$$

$$\Rightarrow g(x) = f(x) + g(x) - f(x) = \sqrt{x+1} - \sqrt{1-x}$$

$$y = \frac{f(x) - r g(x)}{r} = \frac{r \sqrt{x+1} + r \sqrt{1-x} - r \sqrt{x+1} + r \sqrt{1-x}}{r} = \frac{4 \sqrt{1-x}}{4} = \sqrt{1-x}$$

$$D_f = [1, 1] \rightarrow R = [0, \sqrt{r}]$$