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یازدهم تجربی B
تکلیف ۹

$f(x) = \begin{cases} \cot \frac{\pi x}{2} & ; x \leq 1 \\ \sqrt{x^2+1} & ; x > 1 \end{cases}$ $\quad f \circ f(\frac{2}{3}) = ?$

$f(\frac{2}{3}) \to \cot \frac{\frac{2}{3}\pi}{2} \to \cot \frac{\pi}{3} \to \cot 60 \to \frac{\sqrt{3}}{3} \times 3 = \sqrt{3} \to \sqrt{\sqrt{3}^2+1} \to \sqrt{10}$

الف) $g(x) = 2\cos^2 x$ و $f(\frac{1}{x}) = \sqrt{\frac{2x-1}{x^2}}$ $\quad f \circ g(\frac{\pi}{3}) = ?$

$2\cos^2(\frac{\pi}{3}) \to 2(\frac{1}{4}) = \frac{1}{2} \to f \to x = 2 \to \sqrt{\frac{2(2)-1}{4}} \to \frac{\sqrt{3}}{2}$

ب) $g(x) = \frac{x}{1-x}$ و $f(x) = [x]$ $\quad f \circ g(\sqrt{2}) = ?$

$\frac{\sqrt{2}}{1-\sqrt{2}} \to f \to [\frac{\sqrt{2}}{1-\sqrt{2}}] = -1$

$f(x) = \sin x \quad g(x) = x\sqrt{1-x^2} \quad g \circ f(\frac{\pi}{4}) = ?$

$\sin \frac{\pi}{4} \to \frac{\sqrt{2}}{2} \quad \frac{\sqrt{2}}{2}\sqrt{1-(\frac{\sqrt{2}}{2})^2} \to \frac{\sqrt{2}}{2} \times \sqrt{\frac{2}{4}} \to \frac{2}{4} \to \frac{1}{2}$

$f(x) = \{(4,5),(6,5),(8,12),(10,12)\} \quad g(x) = \{(4,6),(2,4),(6,8),(8,10)\}$

الف) $f \circ g(x) \to \{(4,5),(2,5),(6,12),(8,12)\}$

ب) $g \circ f(x) \to \{\emptyset\}$ ؟ $\quad$ ج) $f \circ f(x) = \{\emptyset\}$

د) $g \circ g(x) = \{(4,8),(2,6),(6,10)\}$

$f = \{(2,1),(3,2),(4,5),(1,7)\} \quad g = \{(1,3),(3,1),(a,4),(b,1)\}$

$(4,2) \in f \circ g \quad (4,1) \in g \circ f \quad (a,b) = ?$

$f \circ g = \{(1,1),(3,7),(a,2),(b,7)\} \quad g \circ f = \{(2,3),\}$

$a = 4 \quad b = 5 \quad (4,5)$

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$f(f(x))=4x+3 \quad g(2x+3)=3x-2 \quad gof(-1)$

$x=-1 \to f(f(-1)) \to -1(4)+3 = -1 \to -1 \to x=-2 \to g(-1)=$ (-8)

$f(x)=\sqrt{x+|x|} \geq 0 \quad g(x)=\frac{1}{x^2-4x} \quad D_{gof}$

$D_{gof} \Rightarrow x \in D_f \; ; f(x) \in D_g$

$x^2 \geq 0, \sqrt{2x} \geq 0 \to [0,+\infty)$

$x \in [0,+\infty) \; ; \sqrt{x+|x|} \in R-\{0,4\}$ ($R-\{0,4\}$)

$x<0 \to \sqrt{0}=0$

$x^2-4x \neq 0 \to x(x-4) \neq 0 \to R-\{0,4\}$

$[0,+\infty)$ ∩

$[0,+\infty)-\{0,4\}$

$f(x)=\sqrt{1-x^2} \geq 0 \quad g(x)=\sqrt{x} \geq 0 \quad D_{(f+g)}=[0,1]$

$(f+g)of \quad -1 \leq x \leq 1 \quad [0,+\infty)$

$[0,1]$ ∩

$D_{(f+g)of} \to x \in D_f \; ; f(x) \in D_{(f+g)}$

$0 \leq x \leq 1$

$x \in [-1,1] \; ; 0 \leq \sqrt{1-x^2} \leq 1$

$0 \leq 1-x^2 \leq 1 \Rightarrow -1 \leq -x^2 \leq 0 \to 0 \leq x^2 \leq 1$

الف) $f\left(\frac{3x+1}{x-2}\right)=4x+5 \to \frac{3x+1}{x-2}=t \to 3x+1=tx-2t \to 3x-tx=-1-2t$

$x(3-t)=-1-2t \to x=\frac{-2t-1}{3-t} \to f(t)=4\left(\frac{-2t-1}{3-t}\right)+5$

$f(x)=4\left(\frac{-2x-1}{3-x}\right)+5$

ب) $f\left(x+\frac{1}{x}\right)=x^3+\frac{1}{x^3} \Rightarrow x+\frac{1}{x}=t \to \left(x+\frac{1}{x}\right)^3=x^3+3x+\frac{3}{x}+\frac{1}{x^3}$

$x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right) \to x+\frac{1}{x}=t \to f(t)=t^3-3t$

$f(x)=x^3-3x$

$f(x)=x\sqrt{x} \quad g(f(x)) \Rightarrow g(1)=0 \quad g(2\sqrt{2})=0$

$gof(x) \to$

$f(x)=1 \to x\sqrt{x}=1 \to x=1 \quad$ و $2\sqrt{2}$ $\quad f(x)=2\sqrt{2} \quad x\sqrt{x}=2\sqrt{2} \quad x=2$

$2-1=(1)$