

$$f(n) = \begin{cases} \cot \frac{\pi n}{r} & ; n \leq 1 \\ \sqrt{n^2+1} & ; n > 1 \end{cases} \quad f \circ f\left(\frac{r}{r}\right) \Rightarrow \cot \frac{r\pi}{r} = \cot \pi = \sqrt{3}$$

$$f(\sqrt{r}) = \sqrt{r+1} = (r)$$

۲- الف) $f\left(\frac{1}{n}\right) = \sqrt{\frac{r}{n}-1}$, $g(n) = r \cos n$ $f \circ g\left(\frac{\pi}{r}\right) = ?$

$$g\left(\frac{\pi}{r}\right) = r \times \frac{1}{r} = 1 \quad f\left(\frac{1}{r}\right) \Rightarrow \sqrt{\frac{r}{r}-1} = \sqrt{\frac{r}{r}} = \left(\frac{\sqrt{r}}{r}\right)$$

ب) $f(n) = [n]$ $g(n) = \frac{n}{1-n}$ $f \circ g(\sqrt{r}) = ?$

$$g(\sqrt{r}) = \frac{\sqrt{r}}{1-\sqrt{r}} \times \frac{1-\sqrt{r}}{1-\sqrt{r}} = \frac{\sqrt{r}-r}{-1} = -\sqrt{r}-r$$

$$f(-\sqrt{r}-r) = [-\sqrt{r}-r] = [-r] = (-r)$$

۳- $f(n) = \sin n$ $g(n) = n\sqrt{1-n^2}$ $g \circ f\left(\frac{\pi}{r}\right) = ?$

$$f\left(\frac{\pi}{r}\right) = \sin\left(\frac{\pi}{r}\right) = \frac{\sqrt{r}}{r} \rightarrow g\left(\frac{\sqrt{r}}{r}\right) = \frac{\sqrt{r}}{r} \sqrt{1-\frac{r}{r}} = \frac{\sqrt{r}}{r} \times \sqrt{\frac{r}{r}} = \frac{\sqrt{r}}{r} \times \frac{\sqrt{r}}{r} = \left(\frac{1}{r}\right)$$

۴- $f(n) = \{(1, \omega), (4, \omega), (1, 1), (1, 1)\}$ $g(n) = \{(1, 1), (2, 1), (1, 1), (1, 1)\}$

الف) $f \circ g(n) = \{(1, \omega), (2, \omega), (1, 1), (1, 1)\}$

ب) $g \circ f(n) = \emptyset$ ج) $f \circ f(n) = \emptyset$ د) $g \circ g(n) = \{(1, 1), (2, 1), (1, 1), (1, 1)\}$

۵- $f = \{(1, 1), (2, 2), (1, \omega), (1, \omega)\}$ $g = \{(1, 1), (1, 1), (1, 1), (1, 1)\}$

$(1, 1) \in f \circ g$ $(1, 1) \in g \circ f$ $(a, b) \Rightarrow (1, \omega)$

$$g\left(\frac{f(1)}{a}\right) = 1 \Rightarrow b = a$$

$$f\left(\frac{g(1)}{a}\right) \Rightarrow a = 1$$

۶- $f, g \rightarrow$ تابع خطی $f(f(n)) = rn + r$ $g(rn+r) = rn-r$ $g \circ f(-1) = ?$

$$rn+r=2 \quad n = \frac{2-r}{r} \quad g(z) = \frac{rz-r}{r} \Rightarrow g\left(\frac{2-r}{r}\right) = \frac{r}{r}n - \frac{1}{r}$$

$f(n) = an+b$ $f(an+b) = a(an+b) + b = a^2n + ab + b = a^2n + ab + b = a^2n + (ab+b)$

$a^2 = 1 \rightarrow a = \pm 1$

$ab+b=r \rightarrow \begin{cases} a=1 \rightarrow b=1 \\ a=-1 \rightarrow b=-r \end{cases} \rightarrow f(n) = rn+1 \rightarrow f(-1) = -1$

$g(f(-1)) = g(-1) = \frac{-r}{r} - \frac{1}{r} = (-1)$

۷- $f(n) = \sqrt{n+1}$ $g(n) = \frac{1}{n^2-rn}$ $g \circ f = ?$

$Df = \begin{cases} \sqrt{r}n & n \geq 0 \\ 0 & n < 0 \end{cases} \rightarrow Df = \mathbb{R}$

$Dg \Rightarrow n(n-r) \neq 0 \Rightarrow n \neq 0, r$

$f(n) \neq 0, r \rightarrow \begin{cases} f(n) \neq 0 \rightarrow (0, r) \\ f(n) \neq r \rightarrow \mathbb{R} - \{n\} \end{cases}$

$\Rightarrow D(g \circ f) = (0, r) - \{n\}$

$$f(n) = \sqrt{1-n^2}, \quad g(n) = \sqrt{n}$$

- 1

$$(f+g) \circ f = ?$$

$$\begin{aligned} Df &\rightarrow 1-n^2 \geq 0 \rightarrow -1 \leq n \leq 1 \rightarrow Df = [-1, 1] \\ Dg &\rightarrow n \geq 0 \rightarrow Dg = [0, +\infty) \end{aligned} \quad \left. \vphantom{\begin{aligned} Df &\rightarrow 1-n^2 \geq 0 \rightarrow -1 \leq n \leq 1 \rightarrow Df = [-1, 1] \\ Dg &\rightarrow n \geq 0 \rightarrow Dg = [0, +\infty) \end{aligned}} \right\} D_{f+g} = [0, 1]$$

$$0 \leq \sqrt{1-n^2} \leq 1$$

$$D_{(f+g) \circ f} = [-1, 1]$$

- 1

$$\begin{aligned} \text{الف) } f\left(\frac{r n + 1}{n - r}\right) &= r n + a & \frac{r n + 1}{n - r} = z &\rightarrow z n - r z = r n + 1 \\ f(z) &= f\left(\frac{r z + 1}{z - r}\right) + a = \frac{1 z + r}{z - r} + a \rightarrow f(z) = \frac{r z + 1}{z - r} \\ n &= \frac{r z + 1}{z - r} \\ \frac{r z - 1}{z - r} &= \dots \\ f(n) &= \frac{r n - 1}{n - r} \end{aligned}$$

$$\begin{aligned} \text{ب) } f\left(n + \frac{1}{n}\right) &= n^r + \frac{1}{n} & n + \frac{1}{n} = z &\rightarrow \left(n + \frac{1}{n}\right)^r = n^r + r n + \frac{r}{n} + \frac{1}{n^r} \\ \left(n^r + \frac{1}{n^r}\right) &= \left(n + \frac{1}{n}\right)^r - r\left(n + \frac{1}{n}\right) & n + \frac{1}{n} = z &\rightarrow f(z) = z^r - r z \rightarrow f(n) = n^r - r n \end{aligned}$$

$$\begin{aligned} f(n) = 1 &\rightarrow \sqrt{n} = 1 \\ f(n) = r\sqrt{r} &\rightarrow \sqrt{n} = r\sqrt{r} \\ r = 1 &= 1 \\ \text{انقلاب نقاط متطابق} \end{aligned}$$

- 10