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نام و نام خانوادگی سید علی محمد پاسخنامه تشریحی تکلیف شماره ۹ کلاس B

$$f \circ f\left(\frac{\sqrt{3}}{2}\right) = f\left(f\left(\frac{\sqrt{3}}{2}\right)\right) = f(\sqrt{3}) = 2 \Rightarrow f \circ f\left(\frac{\sqrt{3}}{2}\right) = 2 \quad \checkmark$$

$$f(x) = \begin{cases} \cot \frac{\pi x}{2} & ; x \leq 1 \\ \sqrt{x+1} & ; x > 1 \end{cases} \Rightarrow \begin{cases} f\left(\frac{\sqrt{3}}{2}\right) = \cot \frac{\pi}{6} = \sqrt{3} \\ f(\sqrt{3}) = \sqrt{2} = 2 \end{cases}$$

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$$\text{الف) } f \circ g\left(\frac{\pi}{4}\right) = f\left(g\left(\frac{\pi}{4}\right)\right) = f\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$$

$$f \circ g\left(\frac{\pi}{4}\right) = \frac{\sqrt{3}}{2} \quad \checkmark$$

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$$g(x) = 2 \cos^2 x \Rightarrow g\left(\frac{\pi}{4}\right) = 2 \cos^2 \frac{\pi}{4} = 2 \left(\frac{1}{\sqrt{2}}\right)^2 = 1$$

$$f\left(\frac{1}{2}\right) = \sqrt{\frac{1-1}{2}} = 0 \Rightarrow f\left(\frac{1}{2}\right) = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$$

$$\text{ب) } f \circ g(\sqrt{2}) = f(g(\sqrt{2})) = f(-2-\sqrt{2}) = -2$$

$$g(x) = \frac{x}{1-x} \xrightarrow{x=\sqrt{2}} g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} = \frac{\sqrt{2}(1+\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})} = -(2+\sqrt{2}) = -2-\sqrt{2}$$

$$f(x) = [x] \Rightarrow f(-2-\sqrt{2}) = [-2-\sqrt{2}] = [-3] = -3$$

$$f \circ g(\sqrt{2}) = -2 \quad \checkmark$$

$$g \circ f\left(\frac{\pi}{2}\right) = g\left(f\left(\frac{\pi}{2}\right)\right) = g\left(\frac{\sqrt{2}}{2}\right) = \frac{1}{2}$$

$$f(x) = \sin x \Rightarrow f\left(\frac{\pi}{2}\right) = \frac{\sqrt{2}}{2}$$

$$g(x) = x \sqrt{1-x^2} \Rightarrow g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \sqrt{1-\frac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2} \quad \checkmark$$

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$$\text{الف) } f \circ g(x) = f(g(x)) = \{(1,5), (2,5), (3,12), (4,12)\}$$

$$\begin{array}{l} 1 \rightarrow 2 \rightarrow 5 \quad 2 \rightarrow 2 \rightarrow 5 \\ 3 \rightarrow 1 \rightarrow 12 \quad 4 \rightarrow 1 \rightarrow 12 \end{array}$$

$$\text{ج) } f \circ f(x) = \emptyset \quad \checkmark$$

$$\begin{array}{l} 1 \rightarrow 5 \rightarrow x \\ 2 \rightarrow 5 \rightarrow x \\ 3 \rightarrow 12 \rightarrow x \\ 4 \rightarrow 12 \rightarrow x \end{array}$$

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$$\text{ب) } g \circ f(x) = \emptyset \quad \checkmark$$

$$\begin{array}{l} 1 \rightarrow 5 \rightarrow x \quad 3 \rightarrow 12 \rightarrow x \\ 2 \rightarrow 5 \rightarrow x \quad 4 \rightarrow 12 \rightarrow x \end{array}$$

$$\text{د) } g \circ g(x) = \{(1,8), (2,4), (3,10)\} \quad \checkmark$$

$$\begin{array}{l} 1 \rightarrow 2 \rightarrow 8 \\ 2 \rightarrow 2 \rightarrow 4 \\ 3 \rightarrow 1 \rightarrow 10 \end{array}$$

$$(1,2) \in f \circ g, (1,1) \in g \circ f$$

$$\begin{array}{l} 1 \rightarrow 2 \rightarrow 1 \\ 3 \rightarrow 1 \rightarrow 3 \\ a \rightarrow 2 \rightarrow 2 \\ b \rightarrow 1 \rightarrow 3 \end{array}$$

$$f \circ g(x) = \{(1,1), (2,2), (3,3), (4,4)\}$$

$$\checkmark a=1 \leftarrow (1,2)$$

$$\begin{array}{l} 2 \rightarrow 1 \rightarrow 2 \\ 3 \rightarrow 2 \rightarrow 3 \\ 1 \rightarrow 3 \rightarrow 1 \end{array}$$

$$g \circ f(x) = \{(2,1), (3,2), (1,3), (4,4)\}$$

$$(1,5) = (1,6) \Rightarrow b=5 \quad \checkmark$$

$$(a,b) = (1,5) \quad \checkmark$$

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f تابعی خطی است بصورت $f(x) = ax + b$ بی‌نویس $f(f(x)) = f(ax + b) = a^2x + ab + b = (x+3) \Rightarrow \begin{cases} a^2 \cdot 1 \Rightarrow a = \pm 2 \\ ab + b = 3 \xrightarrow{a=2} 2b = 3 \Rightarrow b = 1.5 \\ a = -2 \Rightarrow -b = 3 \Rightarrow b = -3 \end{cases}$ $f(x) = ax + b \xrightarrow{a=2, b=1.5} f(x) = 2x + 1.5 \xrightarrow{x=1} f(1) = 2.5$ $f(x) = ax + b \xrightarrow{a=-2, b=-3} f(x) = -2x - 3 \xrightarrow{x=1} f(1) = -5$ $g \circ f(-1) = g(f(-1)) = g(-1) = -1$ $g(2x+3) = 3x-2 \Rightarrow g(-1) = -1$ $g \circ f(-1) = -1$ ✓	(2) 6
$D_{g \circ f} = \{x x \in D_f, f(x) \in D_g\}$ $f(x) = \sqrt{x+ x } \Rightarrow x+ x \geq 0 \Rightarrow x \geq -x$ $\Rightarrow D_f = \mathbb{R}$ $g(x) = \frac{1}{x^2-1} \Rightarrow x^2-1 \neq 0 \Rightarrow x(x-1) \neq 0$ $\Rightarrow x \neq 0, 1 \Rightarrow D_g = \mathbb{R} - \{0, 1\}$ $\Rightarrow D_{g \circ f} = (0, +\infty) - \{1\}$ ✓	(2) 7
$D_{(f+g) \circ f} = \{x x \in D_f, f(x) \in D_{f+g}\}$ $x \in [-1, 1]$ $\sqrt{1-x^2} \in [0, 1]$ $\sqrt{1-x^2} \leq 1 \Rightarrow 1-x^2 \leq 1$ $\Rightarrow x^2 \geq 0 \Rightarrow x \in \mathbb{R}$ ✓ $D_{f+g} = D_f \cap D_g \Rightarrow D_{f+g} = [0, 1]$ ✓ $g(x) = \sqrt{x} \Rightarrow x \geq 0 \Rightarrow D_g = [0, +\infty)$ ✓ $f(x) = \sqrt{1-x^2} \Rightarrow 1-x^2 \geq 0 \Rightarrow (1-x)(1+x) \geq 0$ $\xrightarrow{1-x \geq 0, 1+x \geq 0} D_f = [-1, 1]$ ✓ $\Rightarrow D_{(f+g) \circ f} = [-1, 1]$ ✓	(1, 7.5) 8
الف) $f\left(\frac{3x+1}{x-2}\right) = (x+5)$ $\frac{3x+1}{x-2} = t \Rightarrow \frac{3t+1}{t-2} = x$ $\Rightarrow f(t) = \left(\frac{3t+1}{t-2}\right) + 5 = \frac{3t+1}{t-2} + 5$ $= \frac{13t-11}{t-2} \Rightarrow f(x) = \frac{13x-11}{x-2}$ ✓ ب) $f(x, \frac{1}{x}) = x^3 + \frac{1}{x^3} \xrightarrow{x \neq 0} f(t) = t^3 - 2t \Rightarrow f(x) = x^3 - 2x$ ✓ $x^3 + \frac{1}{x^3} = (x, \frac{1}{x})^3 - 2(x)(\frac{1}{x}) \Rightarrow x^3 + \frac{1}{x^3} = (x, \frac{1}{x})^3 - 2(x, \frac{1}{x}) \Rightarrow x^3 + \frac{1}{x^3} = t^3 - 2t$	(2) 9
$g(f(x)) = 0$ $g(1) = 0$ $g(2\sqrt{x}) = 0$ $\Rightarrow \begin{cases} f(x) = 1 = x\sqrt{x} \xrightarrow{1} x^3 = 1 \Rightarrow x = 1 \\ f(x) = 2\sqrt{x} = x\sqrt{x} \xrightarrow{2} x^3 = 8 \Rightarrow x = 2 \end{cases}$ $2-1=1$ ✓	(2) 10