

$$f \circ f\left(\frac{\pi}{4}\right) = f\left(f\left(\frac{\pi}{4}\right)\right) = f(\sqrt{2}) = 2 \Rightarrow f \circ f\left(\frac{\pi}{4}\right) = 2$$

$$f(x) = \begin{cases} \cot \frac{\pi x}{2} & ; x \leq 1 \\ \sqrt{x-1} & ; x > 1 \end{cases} \Rightarrow \begin{cases} f\left(\frac{\pi}{4}\right) = \cot \frac{\pi}{4} = \sqrt{2} \\ f(\sqrt{2}) = \sqrt{2-1} = 1 \end{cases}$$

$$\text{الف) } f \circ g\left(\frac{\pi}{4}\right) = f\left(g\left(\frac{\pi}{4}\right)\right) = f\left(\frac{1}{2}\right) = \frac{\sqrt{2}}{2}$$

$$g(x) = 2 \cos^2 x \Rightarrow g\left(\frac{\pi}{4}\right) = 2 \cos^2 \frac{\pi}{4} = 2 \left(\frac{1}{\sqrt{2}}\right)^2 = 1$$

$$f\left(\frac{1}{2}\right) = \sqrt{\frac{2 \times \frac{1}{2} - 1}{2 \times \frac{1}{2}}} = \sqrt{\frac{1-1}{1}} = 0$$

$$f \circ g\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$\text{ب) } f \circ g(\sqrt{2}) = f(g(\sqrt{2})) = f(-2-\sqrt{2}) = -2$$

$$g(x) = \frac{x}{1-x} \xrightarrow{x=\sqrt{2}} g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} = \frac{\sqrt{2}(1+\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})} = -(2+\sqrt{2}) = -2-\sqrt{2}$$

$$f(x) = [x] \Rightarrow f(-2-\sqrt{2}) = [-2-\sqrt{2}] = -3$$

$$f \circ g(\sqrt{2}) = -3$$

$$g \circ f\left(\frac{\pi}{4}\right) = g\left(f\left(\frac{\pi}{4}\right)\right) = g(\sqrt{2}) = \frac{1}{\sqrt{2}}$$

$$f(x) = \sin x \Rightarrow f\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$g(x) = x \sqrt{1-x^2} \Rightarrow g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \sqrt{1-\frac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$\text{الف) } f \circ g(x) = f(g(x)) = \{(1,5), (2,5), (3,12), (4,12)\}$$

$$\begin{array}{l} 1 \rightarrow 2 \rightarrow 5 \quad 2 \rightarrow 3 \rightarrow 5 \\ 3 \rightarrow 4 \rightarrow 12 \quad 4 \rightarrow 10 \rightarrow 12 \end{array}$$

$$\text{ج) } f \circ f(x) = \emptyset$$

$$\begin{array}{l} 1 \rightarrow 5 \rightarrow x \\ 2 \rightarrow 5 \rightarrow x \\ 3 \rightarrow 12 \rightarrow x \\ 4 \rightarrow 12 \rightarrow x \end{array}$$

$$\text{ب) } g \circ f(x) = \emptyset$$

$$\begin{array}{l} 1 \rightarrow 5 \rightarrow x \quad 3 \rightarrow 12 \rightarrow x \\ 2 \rightarrow 5 \rightarrow x \quad 4 \rightarrow 12 \rightarrow x \end{array}$$

$$\text{د) } g \circ g(x) = \{(1,8), (2,4), (3,10)\}$$

$$\begin{array}{l} 1 \rightarrow 2 \rightarrow 8 \quad 3 \rightarrow 4 \rightarrow 10 \\ 2 \rightarrow 3 \rightarrow 4 \quad 4 \rightarrow 10 \rightarrow x \end{array}$$

$$(1,2) \in f \circ g, (1,1) \in g \circ f$$

$$\begin{array}{l} 1 \rightarrow 2 \rightarrow 1 \\ 3 \rightarrow 1 \rightarrow 3 \\ 4 \rightarrow 2 \rightarrow 2 \\ 2 \rightarrow 1 \rightarrow 3 \end{array}$$

$$f \circ g(x) = \{(1,1), (2,3), (3,2), (4,3)\}$$

$$a=1 \leftarrow (1,2)$$

$$\begin{array}{l} 2 \rightarrow 1 \rightarrow 2 \\ 3 \rightarrow 2 \rightarrow 3 \\ 4 \rightarrow 3 \rightarrow 1 \\ 1 \rightarrow 3 \end{array}$$

$$g \circ f(x) = \{(2,1), (3,2), (4,3), (1,3)\}$$

$$(1,3) = (1,3) \Rightarrow b=3$$

$$(a,b) = (1,3)$$

f تابعی خطی است بصورت $f(x) = ax + b$ می نویسیم $f(f(x)) = f(ax + b) = a^2x + ab + b = (x+3) \Rightarrow \begin{cases} a^2 = 1 \Rightarrow a = \pm 1 \\ ab + b = 3 \xrightarrow{a=1} 2b = 3 \Rightarrow b = 1.5 \\ \xrightarrow{a=-1} -b = 3 \Rightarrow b = -3 \end{cases}$ $f(x) = ax + b \xrightarrow{a=1, b=1.5} f(x) = x + 1.5 \xrightarrow{x=-1} f(-1) = -1$ $\xrightarrow{a=-1, b=-3} f(x) = -x - 3 \xrightarrow{x=-1} f(-1) = -1$ $g \circ f(-1) = g(f(-1)) = g(-1) = -1$ $g(2x+3) = 3x-1 \Rightarrow g(-1) = -1$ $g \circ f(-1) = -1$	6
$D_{g \circ f} = \{x x \in D_f ; f(x) \in D_g\}$ $x \in \mathbb{R}$ $\begin{cases} \sqrt{x+ x } \neq 0 \\ \Rightarrow x+ x \neq 0 \Rightarrow x \neq -x \\ \Rightarrow x > 0 \\ \sqrt{x+ x } \neq 1 \\ \Rightarrow x+ x \neq 1 \Rightarrow \frac{0}{1 \neq 0} \neq 1 \Rightarrow x \neq 1 \end{cases}$	$f(x) = \sqrt{x+ x } \Rightarrow x+ x \geq 0 \Rightarrow x \geq -x$ $\Rightarrow D_f = \mathbb{R}$ $g(x) = \frac{1}{x^2-1} \Rightarrow x^2-1 \neq 0 \Rightarrow x(x-1) \neq 0$ $\Rightarrow x \neq 0, 1 \Rightarrow D_g = \mathbb{R} - \{0, 1\}$ $\Rightarrow D_{g \circ f} = (0, +\infty) - \{1\}$ 7
$D_{(f+g) \circ f} = \{x x \in D_f, f(x) \in D_{f+g}\}$ $x \in [-1, 1]$ $\begin{cases} \sqrt{1-x^2} \in [0, 1] \\ \sqrt{1-x^2} \leq 1 \Rightarrow 1-x^2 \leq 1 \\ \Rightarrow x^2 \geq 0 \Rightarrow x \in \mathbb{R} \end{cases}$	$D_{f+g} = D_f \cap D_g \Rightarrow D_{f+g} = [0, 1]$ $g(x) = \sqrt{x} \Rightarrow x \geq 0 \Rightarrow D_g = [0, +\infty)$ $f(x) = \sqrt{1-x^2} \Rightarrow 1-x^2 \geq 0 \Rightarrow (1-x)(1+x) \geq 0$ $\xrightarrow{1-x \geq 0} D_f = [-1, 1]$ $\Rightarrow D_{(f+g) \circ f} = [0, 1]$ 8
الف) $f\left(\frac{3x+1}{x-2}\right) = (x+5)$ $\frac{3x+1}{x-2} = t \Rightarrow \frac{3t+1}{t-2} = x$ $\Rightarrow f(t) = \left(\frac{3t+1}{t-2}\right) + 5 = \frac{3t+1}{t-2} + 5$ $= \frac{13t-11}{t-2} \Rightarrow f(x) = \frac{13x-11}{x-2}$ ب) $f(x, \frac{1}{x}) = x^3 + \frac{1}{x^3} \xrightarrow{x=\frac{1}{x}} f(t) = t^3 - 2t \Rightarrow f(x) = x^3 - 2x$ $x^3 + \frac{1}{x^3} = (x, \frac{1}{x})^3 - 2(x)(\frac{1}{x}) \Rightarrow x^3 + \frac{1}{x^3} = (x, \frac{1}{x})^3 - 2(x, \frac{1}{x}) \Rightarrow x^3 + \frac{1}{x^3} = t^3 - 2t$	9
$\left. \begin{matrix} g(f(x)) = 0 \\ g(1) = 0 \\ g(2\sqrt{2}) = 0 \end{matrix} \right\} \Rightarrow \begin{cases} f(x) = 1 = x\sqrt{x} \xrightarrow{1} x^3 = 1 \Rightarrow x = 1 \\ f(x) = 2\sqrt{2} = x\sqrt{x} \xrightarrow{2} x^3 = 8 \Rightarrow x = 2 \end{cases}$ $2-1=1$ 10	