

$$f(x) = \begin{cases} \cot \frac{\pi x}{r} ; x \leq 1 \\ \sqrt{x^2 + 1} ; x > 1 \end{cases} \quad f\left(\frac{r}{r}\right) = \cot \frac{\pi}{r} \Rightarrow \frac{\pi}{r} = \frac{\pi}{2} \Rightarrow \cot\left(\frac{\pi}{2}\right) = 0$$

$$f(\sqrt{r}) = \sqrt{r} = \boxed{r}$$

$$\text{الف) } f\left(\frac{1}{x}\right) = \sqrt{\frac{r^2 x^2 - 1}{x^2}} \quad \frac{1}{x} = + \frac{r^2 x^2 - 1}{x^2} - \frac{r^2}{x^2} = -\frac{1}{x^2} \rightarrow \frac{r^2 x^2 - 1}{x^2} = r^2 - \frac{1}{x^2}$$

$$f(x) = \sqrt{r^2 x^2 - 1} \leftarrow f\left(\frac{1}{x}\right) = \sqrt{r^2 - \frac{1}{x^2}}$$

$$g(x) = r \cos x$$

$$\hookrightarrow g\left(\frac{\pi}{r}\right) = \frac{1}{r} \rightarrow f\left(\frac{1}{r}\right) = \sqrt{1 - \frac{1}{r^2}} = \frac{\sqrt{r^2 - 1}}{r}$$

$$\text{ب) } f(x) = [x] \quad g(x) = \frac{x}{1-x} \quad g(\sqrt{r}) = \frac{\sqrt{r} \times (1+\sqrt{r})}{1-\sqrt{r} \times (1+\sqrt{r})} = \frac{\sqrt{r} + r}{1-r} = -\sqrt{r} - r$$

$$f(-\sqrt{r} - r) = [-\sqrt{r} - r] = -r$$

$$f(x) = \sin x, \quad g(x) = x \sqrt{1-x^2}$$

$$f\left(\frac{r}{r}\right) = \frac{\sqrt{r}}{r} \rightarrow g\left(\frac{\sqrt{r}}{r}\right) = \frac{\sqrt{r}}{r} \times \sqrt{1 - \frac{1}{r}} = \frac{\sqrt{r}}{r} \times \frac{1}{\sqrt{r}} = \boxed{\frac{1}{r}}$$

$$g(x) = \{(f, 2), (2, f), (f, 1), (1, f)\}$$

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$$\text{الف) } f \circ g(x) = \{(f, 2), (2, f), (1, f), (f, 1)\}$$

$$\text{ب) } g \circ f(x) = \emptyset$$

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$$\text{د) } g \circ g(x) = \{(f, 1), (1, f), (2, f), (f, 2)\}$$

$$f = \{(2, 1), (1, 2), (f, 2), (2, f)\} \quad g = \{(1, 2), (2, 1), (a, r), (b, 1)\}$$

$$(f, 2) \in f \circ g \rightarrow g(f) = 2 \rightarrow f(2) = 2 \rightarrow \boxed{a = f}$$

$$(f, 1) \in g \circ f \rightarrow f(f) = a \rightarrow g(a) = 1 \rightarrow \boxed{b = a} \quad (f, a)$$

$$f(f(x)) = fx + r \quad x + b$$

$$f(f(x)) = a^2(x+b) + b \quad a^2x + ab + b = fx + r \rightarrow \boxed{a = \pm r}$$

$$f(f(x)) = a^2x + ab + b \quad \text{I) } a = r \rightarrow b = 1 \rightarrow f(x) = rx + 1$$

$$g(rx + 1) = rx - r \quad \text{II) } a = -r \rightarrow b = -r \rightarrow -rx - r$$

$$f(-1) = -1 \rightarrow rx + r = -1 \rightarrow \boxed{x = -r} \rightarrow g(-1) = \boxed{-1}$$

وی در هر صورت آنه بیوی ایس

۱- یکبار به یوا منفی یک ی شوی

$$f(x) = \sqrt{x+|x|}, \quad g(x) = \frac{1}{x^2 - 5x} \rightarrow 0, \quad D_{g \circ f} = ?$$

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$$D_{g \circ f} = \{x \in D_f, f(x) \in D_g\}$$

$$D_f = x + |x| \geq 0$$

$$D_f = \mathbb{R}$$

$$[x \in \mathbb{R}, f(x) \in f - \{0, 5\}] \rightarrow D_{g \circ f} = \mathbb{R} - \{0, 5\}$$

$$D_f = \mathbb{R} - \{0, 5\}$$

$$f(x) = \sqrt{1-x^2}, \quad g(x) = \sqrt{x}, \quad D_{(f+g) \circ f} = ?$$

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$$D_{(f+g) \circ f} = [x \in f, f(x) \in D_{(f+g)}] \quad \begin{cases} 1-x^2 \geq 0 \\ 1 \geq x^2 \rightarrow 1 \geq x \geq -1 \end{cases}$$

$$[x \in [1, -1], f(x) \in D_{(f+g)}]$$

$$\sqrt{1-x^2} + \sqrt{x}$$

$$f\left(\frac{r_{n+1}}{n-r}\right) = f_n + d \quad \frac{r_{n+1}}{n-r} = a \quad n = \frac{a+1}{a-r}$$

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$$f(a) = f\left(\frac{a+1}{a-r}\right) + d \rightarrow f(a) = \frac{a+1}{a-r} + d \rightarrow \frac{f(a) + d(a-1)}{a-r} = \frac{a+1}{a-r}$$

$$f(x) = \frac{ax-1}{x-r}$$

$$f\left(x + \frac{1}{n}\right) = x^r + \frac{1}{n^r} \quad x + \frac{1}{n} = a \quad f(a) = a^r - r_n$$

$$\left(x + \frac{1}{n}\right)^r = x^r + \frac{1}{n^r} + r\left(x + \frac{1}{n}\right) \quad f(x) = x^r - r_n$$

$$x^r + \frac{1}{n^r} = \frac{\left(x + \frac{1}{n}\right)^r - r\left(x + \frac{1}{n}\right)}{a^r - r_n}$$

$$g \circ f(x) = 0$$
$$y=0 \begin{cases} \rightarrow x = \sqrt{2} \rightarrow x\sqrt{x} = \sqrt{2} \rightarrow x = 2 \\ \rightarrow x = 1 \rightarrow x\sqrt{x} = 1 \rightarrow x = 1 \end{cases}$$