

$$f(x) = \begin{cases} \cot \frac{\pi x}{4} & ; x \leq 1 \\ \sqrt{x^2+1} & ; x > 1 \end{cases}$$

$$f \circ f\left(\frac{1}{\sqrt{3}}\right) = f\left(f\left(\frac{1}{\sqrt{3}}\right)\right) = f(\sqrt{3}) = \sqrt{3+1} = 2 \quad \checkmark$$

$$f\left(\frac{1}{\sqrt{3}}\right) = \cot \frac{\pi \times \frac{1}{\sqrt{3}}}{4} = \cot \frac{\pi}{4} = \sqrt{3} \approx 1.732$$

الف) $g(x) = 2 \cos^2 x$, $f\left(\frac{1}{x}\right) = \sqrt{\frac{2x-1}{x^2}}$

$$f \circ g\left(\frac{\pi}{4}\right) = f\left(g\left(\frac{\pi}{4}\right)\right) = f\left(\frac{1}{2}\right) = \sqrt{\frac{2 \times \frac{1}{2} - 1}{\left(\frac{1}{2}\right)^2}} = \sqrt{\frac{0}{\frac{1}{4}}} = 0 \quad \checkmark$$

ب) $g(x) = \frac{x}{1-x}$, $f(x) = [x]$ $\rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \xrightarrow{\sqrt{2} \approx 1.414} \frac{1.414}{1-1.414} = -3.414$

$$f \circ g(\sqrt{2}) = f(g(\sqrt{2})) = f(-3.414) = [-3.414] = -4 \quad \checkmark$$

$f(x) = \sin x$ / $g(x) = x \sqrt{1-x^2}$

$$g \circ f\left(\frac{\pi}{4}\right) = g\left(f\left(\frac{\pi}{4}\right)\right) = g\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}} \left(\sqrt{1 - \left(\frac{1}{\sqrt{2}}\right)^2} \right) = \frac{1}{\sqrt{2}} \left(\sqrt{\frac{1}{2}} \right) = \frac{1}{2} \quad \checkmark$$

$$f\left(\frac{\pi}{4}\right) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} \approx \frac{1}{\sqrt{2}} \quad \checkmark$$

$g(x) = \{(x, y), (y, x), (y, 1), (1, y)\}$ / $f(x) = \{(x, a), (y, a), (1, y), (1, y, 1)\}$

الف) $f \circ g(x) = \{(x, a), (y, a), (y, 1), (1, y)\} \quad \checkmark$

ب) $g \circ f(x) = \emptyset \quad \checkmark$

ج) $f \circ f(x) = \emptyset \quad \checkmark$

د) $g \circ g(x) = \{(x, 1), (y, y), (y, 1), (1, y)\} \quad \checkmark$

$f = \{(x, 1), (y, 2), (x, a), (1, y)\}$, $g = \{(1, y), (y, 1), (a, y), (b, 1)\}$

$(x, 2) \in f \circ g \rightarrow f(g(x)) = 2 \rightarrow g(x) = y \rightarrow x = 1 \quad \checkmark$

$(x, 1) \in g \circ f \rightarrow g(f(x)) = 1 \rightarrow f(x) = b \rightarrow x = a \quad \checkmark$

$$f(f(n)) = f(n+1)$$

$$f(n) = an+b \rightarrow f(ax+b) = a^2n+ab+b = f(n+1) \begin{cases} a=1, b=1 \rightarrow f(n) = n+1 \\ a=-1, b=-1 \rightarrow f(n) = -n-1 \\ f(-1) = -1 \end{cases}$$

$$g \circ f(-1) = g(f(-1)) = g(-1) = -1$$

$$g(xn+1) = xn-1 \rightarrow \begin{cases} xn+1 = -1 \\ x = -2 \end{cases} \rightarrow g(-1) = -1$$

$$Dg \circ f = \{x \in Df : f(x) \in Dg\} = Dg \circ f = (0, +\infty) - \{1\}$$

$$g(n) = \frac{1}{n^2-1} \rightarrow Dg = \mathbb{R} - \{-1, 1\}$$

$$f(n) = \sqrt{n+1} \rightarrow \begin{cases} \sqrt{n} & ; n \geq 0 \\ 0 & ; n < 0 \end{cases} \rightarrow \begin{cases} \sqrt{n} \geq 0 \rightarrow n \geq 0 \\ \sqrt{n} > 0 \rightarrow n > 0 \end{cases}$$

$$Df = \mathbb{R}$$

$$Dg = \mathbb{R} - \{0, 1\} \rightarrow f(n) \neq 0 \rightarrow n \neq (-\infty, 0] \Rightarrow n \neq (-\infty, 0] \cup \{1\}$$

$$f(n) \neq 1 \rightarrow \sqrt{n+1} \rightarrow n \neq 0$$

$$D(f+g) \circ f / g(n) = \sqrt{n} \quad f(n) = \sqrt{1-n^2} \rightarrow Df \Rightarrow 1-n^2 > 0 \rightarrow -1 \leq n \leq 1$$

$$Dg = [0, +\infty)$$

$$D(f+g) \circ f = \{x \in Df : f(x) \in Dg\} = [-1, 1]$$

$$Df = [-1, 1] \cap Dg = [0, 1]$$

$$Df = [-1, 1] \quad Dg = [0, +\infty) \rightarrow D(f+g) \circ f = [0, 1]$$

$$f\left(\frac{n+1}{n-1}\right) = f(n+1)$$

$$\frac{n+1}{n-1} = t \rightarrow n = \frac{t+1}{t-1} \rightarrow f(t) = f\left(\frac{t+1}{t-1}\right) + \omega = \frac{1+t+\omega t-1}{t-1} = \frac{1+t-1}{t-1} = \frac{t}{t-1}$$

$$f(n) = \frac{1+n-1}{n-1} = \frac{n}{n-1}$$

$$f\left(n+\frac{1}{n}\right) = n^3 + \frac{1}{n^3} = \left(n+\frac{1}{n}\right)^3 - 3\left(n+\frac{1}{n}\right)$$

$$n+\frac{1}{n} = t \rightarrow f(t) = t^3 - 3t \Rightarrow f(n) = n^3 - 3n$$

$$g(1) = 0 \quad f(n) = n\sqrt{n}$$

$$g(x\sqrt{x}) = 0 \rightarrow f(n) = 1 \rightarrow x\sqrt{n} = 1 \rightarrow n = \frac{1}{x^2}$$

$$g \circ f(n) = 0 \rightarrow g(f(n)) = 0 \rightarrow f(n) = x\sqrt{x} \rightarrow x\sqrt{n} = x\sqrt{x} \rightarrow \sqrt{n} = \sqrt{x} \rightarrow n = x$$

$$(x\sqrt{x}-n) = 0 \rightarrow x\sqrt{x} = n$$