

$$f(x) = \begin{cases} \cot \frac{\pi x}{x} & x < 1 \\ \sqrt{x^2 + 1} & x > 1 \end{cases} \rightarrow f\left(\frac{1}{x}\right) = \cot\left(\frac{\pi}{x}\right) = \cot\left(\frac{\pi}{12}\right) = \cot\left(\frac{\pi}{6}\right)$$

$$\rightarrow \frac{\cos\left(\frac{\pi}{6}\right)}{\sin\left(\frac{\pi}{6}\right)} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

$$\rightarrow f(\sqrt{3}) = \sqrt{(\sqrt{3})^2 + 1} = \sqrt{3+1} = \sqrt{4} = 2$$

$$\text{الف) } f\left(\frac{1}{x}\right) = \sqrt{\frac{x^2-1}{x^2}}, g(x) = 2\cos^2 x \rightarrow f \circ g\left(\frac{\pi}{4}\right)$$

$$\rightarrow g\left(\frac{\pi}{4}\right) = g\left(\frac{\pi}{4}\right) = 2\cos^2\left(\frac{1}{4}\right) = 2 \times \left(\frac{1}{2}\right)^2 = \frac{1}{2}$$

$$\rightarrow f\left(\frac{1}{x}\right) = f\left(\frac{1}{\frac{1}{2}}\right) = \sqrt{\frac{4-1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

$$\text{ب) } f(x) = [x], g(x) = \frac{x}{1-x} \rightarrow \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+1}{1-2} = -\sqrt{2}-1$$

$$\rightarrow [-\sqrt{2}-1] = [-2]$$

$$f(x) = \sin x, g(x) = x\sqrt{1-x^2} \rightarrow g \circ f\left(\frac{\pi}{4}\right) \rightarrow$$

$$f(x) = f\left(\frac{\pi}{4}\right) = f\left(\frac{\pi}{4}\right) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\rightarrow g(x) = g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \sqrt{1-\left(\frac{\sqrt{2}}{2}\right)^2} = \frac{\sqrt{2}}{2} \sqrt{1-\frac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{2}{4} = \frac{1}{2}$$

$$\text{الف) } f \circ g(x) = \{(f, d), (e, 12), (r, d), (18, 12)\}$$

$$\text{ب) } g \circ f(x) = \emptyset$$

$$\text{ج) } f \circ f(x) = \emptyset$$

$$\text{د) } g \circ g(x) = \{(f, 18), (r, 6), (e, 10)\}$$

$$f = \{(r, 1), (r, 2), (f, d), (1, v)\} \quad g = \{(b, 2), (3, 1), (a, 3) \text{ و } (b, 1)\}$$

$$(f, 2) \in f \circ g \rightarrow f(g(f)) = 2, a = f \rightarrow g(f) = 3 \rightarrow \boxed{a = 3}$$

$$(f, 1) \in g \circ f \rightarrow g(d) = 1 \rightarrow (d, 1) \rightarrow \boxed{b = d}$$

$$\rightarrow (a, b) = (f, d)$$

