نام و نام خانوادگی ...امیرحامد جنابی... پاسخنامه تشریحی تکلیف شماره ...۹... کلاس ...یازدهم تجربی B...

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$f(x) = \begin{cases} \cot \frac{\pi x}{4} & ; x \le 1 \\ \sqrt{x^2+1} & ; x > 1 \end{cases}$ $\rightarrow$ $f(f(\frac{2}{3})) = ?$

$f(x) = \cot \frac{\pi x}{4} \xrightarrow{x=\frac{2}{3}} \cot \frac{2\pi}{12} = \cot \frac{\pi}{6} = \sqrt{3}$

$f(\frac{2}{3}) = \sqrt{3}$ (زاویه ۳۰°)

$f(f(\frac{2}{3})) \rightarrow f(\frac{2}{3}) \rightarrow \frac{2}{3} \le 1$

$f(\sqrt{3}) \xrightarrow{\sqrt{3}>1} \sqrt{(\sqrt{3})^2+1} = \sqrt{4} = 2 \rightarrow fof(\frac{2}{3}) = 2$

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الف) $g(x) = 2\cos^2 x$ $\quad f(x) = \sqrt{\frac{2x-1}{x^2}}$ $\quad f(g(\frac{\pi}{3})) =$

$g(\frac{\pi}{3}) = 2\cos^2\frac{\pi}{3} = 2(\frac{1}{2})^2 = \frac{2}{4} = \frac{1}{2}$

$f(\frac{1}{2}) = \sqrt{\frac{2(2)-1}{4}} = \frac{\sqrt{3}}{2}$ $\quad fog(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$

ب) $f(x) = [x]$ $\quad f(g(\sqrt{2})) = \rightarrow f(-\sqrt{2}-2) = [-\sqrt{2}-2] = [-\sqrt{2}] - 2$

$-\sqrt{2} = -1.4...$ $\rightarrow -2-2 = \boxed{-4}$

$g(x) = \frac{x}{1-x}$ $\rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \rightarrow \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{1-2} = -\sqrt{2}-2$

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$f(x) = \sin x \xrightarrow{x=\frac{\pi}{4}} f(\frac{\pi}{4}) = \sin\frac{\pi}{4} \rightarrow f(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$g(x) = x\sqrt{1-x^2}$

$g(f(\frac{\pi}{4})) \xrightarrow{f(\frac{\pi}{4})=\frac{\sqrt{2}}{2}} g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2}\sqrt{1-(\frac{\sqrt{2}}{2})^2} = \frac{\sqrt{2}}{2}\sqrt{1-\frac{2}{4}} = \frac{\sqrt{2}}{2}\sqrt{\frac{2}{4}} = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2}$

$= \frac{2}{4} = \frac{1}{2} \rightarrow gof(\frac{\pi}{4}) = \frac{1}{2}$

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الف) $f(g(x)) = \{(4,5),(2,5),(6,12),(8,12)\}$

ب) $g(f(x)) : R_f \notin D_g$ تابع تهی است

ج) $f(f(x)) : R_f \notin D_f$ تابع تهی است

د) $g(g(x)) = \{(4,8),(2,6),(6,10)\}$

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$f(x) = \{(2,2),(3,2),(4,5),(1,7)\}$ $\quad [a,b] = (4,5)$

$g(x) = \{(1,2),(3,1),(a,3),(b,1)\}$

$fog \ni (4,2)$ $\quad f(g(x)) \ni (4,2) \xrightarrow[y=2]{x=4} f(g(4)) = 2$, $f(3) = 2$, $f(g(4)) = f(3)$, $g(4) = 3$

$f(4) = 5$

$gof \ni (4,1)$ $\quad g(f(x)) \ni (4,1) \xrightarrow[y=1]{x=4} g(f(4)) = 1$, $g(a) = 3$, $g(4) = g(a)$, $\boxed{4 = a}$

$g(5) = 1$, $g(b) = 1$ $\}$ $g(5) = g(b)$, $\boxed{b = 5}$

$f(f(x)) = 4x+3$ $f(x)=ax+b \xrightarrow{x \to ax+b} a(ax+b)+b = fof$ $a^2x+ba+b = 4x+3 \leftarrow \begin{cases} f(f(x)) = a^2x+ba+b \\ f(f(x)) = 4x+3 \end{cases}$ $a^2=4$, $+2 \to 4x+3b=4x+3$ $b=1 \to f(x)=2x+1$ I $a=\pm 2$, $-2 \to 4x-b=4x+3 \to f(x)=-2x-3$ II $g(2x+3)=3x-2$ I, II $= gof(-1) = \frac{3x+3-13}{2} = -8$ $g(x)=\frac{3x-13}{2}$ $b=-3$ $g(f(x))=$ → II $= gof(-1)$ $\frac{-6x-9-13}{2} = \frac{16}{2}$ دو جواب باطل است ، $gof(-1)=-8$	۶
$f(x)=\sqrt{x+\lvert x \rvert}$ $D_{gof} = (0,+\infty) - \{8\}$ $g(x) = \frac{1}{x^2-4x} \xrightarrow{x \to \sqrt{x+\lvert x \rvert}} \frac{1}{(\sqrt{x+\lvert x \rvert})^2 - 4(\sqrt{x+\lvert x \rvert})}$ I $= x \neq 8$ II $= x>0$ I∩II صفر نباید باشد ، و [illegible] $g(f(x)) \Rightarrow g(f(x)) = \frac{1}{(\sqrt{x+\lvert x \rvert})^2 - 4(\sqrt{x+\lvert x \rvert})}$ $D_{fof} = \sqrt{x+\lvert x \rvert} \geq 0$ $x>0 \to 2x \geq 0$ $x \geq 0$ ✓ (I) $x+\lvert x \rvert \geq 0$ $x<0$ $x+\lvert x \rvert = 0$ ✗ II $= (\sqrt{x+\lvert x \rvert})^2 - 4(\sqrt{x+\lvert x \rvert}) \neq 0$ $\sqrt{x+\lvert x \rvert}(\sqrt{x+\lvert x \rvert}-4) \neq 0$ $x \neq 0$ ✓ $\sqrt{x+\lvert x \rvert} \neq 4$ $x+\lvert x \rvert \neq 16$ $x \neq 8$	۷
$g(x)=\sqrt{x}$, $f(x)=\sqrt{1-x^2}$ $D_g \cap D_f = [0,1] = D_{f+g}$ $D_g=[0,+\infty)$ $D_f=[0,1]$ $D_{(f+g)of} = \{x \in D_f , f(x) \in D_{f+g}\}$ $f+g(f(x)) \to$ $f(x)=\sqrt{1-x^2} \to R_f=[0,1]$ $D_{(f+g)of} = \{x \in [-1,1], [0,1] \in [0,1]\}$ ✓ $D_f=[-1,1]$ $D_{(f+g)of} = [-1,1]$	۸
ب) $f(x+\frac{1}{x}) = x^3+\frac{1}{x^3}$ $x+\frac{1}{x}=t$ $(x+\frac{1}{x})^3$ $f(x+\frac{1}{x}) = x^3+\frac{1}{x^3}+3x^2\frac{1}{x}+3x\frac{1}{x^2}-3x^2\frac{1}{x}-3x\frac{1}{x^2}$ $f(x+\frac{1}{x}) = (x+\frac{1}{x})^3 - 3(x+\frac{1}{x}) \Rightarrow$ $f(x)=x^3-3x$ الف) $\frac{3x+1}{x-2}=t$ $tx-2t=3x+1$ $tx-3x=2t+1$ $x(t-3)=2t+1 \to x=\frac{2t+1}{t-3}$ $f(t) = 4(\frac{2t+1}{t-3})+5$ $f(x) = 4(\frac{2x+1}{x-3})+5$	۹
$g(1)=0$ (I) $g(2\sqrt{2})=0$ (II) $f(x)=x\sqrt{x}$ (III) $g(f(x))=0$ I → $f(x)=1 \xrightarrow{III} x\sqrt{x}=1 \to x^3=1 \to x=1$ II → $f(x)=2\sqrt{2} \to x\sqrt{x}=2\sqrt{2} \to x^3=8 \Rightarrow x=2$ $gof(1)=0$, $gof(2)=0$ $2-1=1$ $1-2=-1$	۱۰