

$$\cos \frac{\pi}{4} \times \frac{2}{2} = \cos \frac{\pi}{4} = \sqrt{2} = f\left(\frac{2}{2}\right)$$

$$f(\sqrt{2}) = \sqrt{x^2 + 1} = \sqrt{2 + 1} = \sqrt{3} = 3 \checkmark$$

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$$g\left(\frac{\pi}{2}\right) = 2 \cos^2 \frac{\pi}{2} = 2 \left(\frac{1}{2}\right)^2 = \frac{1}{2}$$

$$f\left(\frac{1}{2}\right) = \sqrt{\frac{2(2)}{2^2} - 1} = \sqrt{\frac{2}{2}} = \frac{\sqrt{2}}{2} \checkmark$$

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$$g(\sqrt{2}) = \frac{\sqrt{2}}{1 - \sqrt{2}} = \frac{\sqrt{2}(1 + \sqrt{2})}{1 - 2} = -\sqrt{2}(1 + \sqrt{2}) = -2 - \sqrt{2}$$

$$f(-2 - \sqrt{2}) = [-2 - \sqrt{2}] = -4 \checkmark$$

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$$f\left(\frac{\pi}{2}\right) = \sin \frac{\pi}{2} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$g\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}} \sqrt{1 - \left(\frac{1}{\sqrt{2}}\right)^2} = \frac{1}{\sqrt{2}} \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2} \checkmark$$

$$g \circ f = \emptyset \checkmark$$

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$$\{(2, 5)(6, 12)(8, 12)(2, 5)\} \checkmark$$

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$$\{(2, 8)(2, 4)(4, 1)\} \checkmark$$

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$$f \circ f = \emptyset \checkmark$$

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$$f(g(n)) = 2 \Rightarrow g(n) = 2 \Rightarrow n = a, f \checkmark$$

$$t \in f \Rightarrow f(t) = 2 \Rightarrow g(f(t)) = g(2) = 1 \Rightarrow b = 2 \checkmark$$

$$\Rightarrow (a, b) = (2, 2) \checkmark$$

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$$g(rn+r) = rn+r, a(rn+r)+b = ran+ra+b \Rightarrow a = \frac{r}{r} \Rightarrow g(n) = \frac{r}{r}n = \frac{r}{r} \quad \begin{matrix} b = -\frac{1r}{r} \end{matrix} \quad \gamma$$

$$f(f(n)) = f(n+r) = a(f(n)) + b = a(an+b) + b = a^2n + ab + b \Rightarrow \begin{cases} a^2r \Rightarrow b=1 \\ a(-r) \Rightarrow b=-r \end{cases} \quad (2)$$

$$\Rightarrow f(n) = \begin{cases} rn+1 \Rightarrow f(-1) = -r+1 = -1 \\ -rn-r \Rightarrow f(-1) = r-r = -1 \end{cases} \Rightarrow g(-1) = -\frac{r}{r} - \frac{1r}{r} = -1 \quad \checkmark$$

$$f(n) \in D_g \quad n \cdot r - rn = n(n-r) \neq 0 \Rightarrow n \neq 0 \quad D_g = \mathbb{R} - \{0, r\} \quad \checkmark$$

$$n \cdot n \in D_f = \mathbb{R} \Rightarrow \text{red } n > 0 \quad \Rightarrow f(n) \neq \begin{cases} c \Rightarrow n+|n| \neq 1 \Rightarrow n \neq \frac{1}{2} \end{cases} \Rightarrow D_f = \mathbb{R} - \{0, \frac{1}{2}\}$$

$$n \in D_f \Rightarrow n \in \mathbb{R} - \{0, \frac{1}{2}\} \quad (0, +\infty) - \{\frac{1}{2}\}$$

$$f(n) \in D_{f+g} \quad (f+g)(n) = \sqrt{n} + \sqrt{1-n} < \frac{1-n^2}{n} \quad \begin{matrix} 1-n^2 \geq 0 \Rightarrow n^2 \leq 1 \Rightarrow -1 \leq n \leq 1 \quad \textcircled{1} \\ n > 0 \end{matrix} \quad \wedge \quad (1, \sqrt{2}) \quad \checkmark$$

$$\textcircled{1} \wedge \textcircled{2} \Rightarrow \begin{matrix} -1 \leq n \leq 1 \\ n > 0 \end{matrix} = D_{f+g} \Rightarrow \begin{matrix} 0 \leq f(n) \leq 1 \\ \Rightarrow 0 \leq \sqrt{1-n^2} \leq 1 \Rightarrow 0 \leq 1-n^2 \leq 1 \\ \Rightarrow 1-n^2 \geq 0 \Rightarrow n^2 \leq 1 \Rightarrow -1 \leq n \leq 1 \end{matrix} \quad \textcircled{3}$$

$$-1 \leq n^2 \leq 0 \Rightarrow 0 \leq n^2 \leq 1 \Rightarrow -1 \leq n \leq 1 \quad \textcircled{4}$$

$$\textcircled{3} \wedge \textcircled{4} \Rightarrow -1 \leq n \leq 1 \quad [-1, 1] \quad n \in D_f \Rightarrow -1 \leq n \leq 1$$

$$\frac{rn+1}{n-r} = t \Rightarrow n = \frac{-rt-1}{-t+r} \Rightarrow (n+r) = \frac{-1t-r-2t+10}{-t+r} = \frac{-1^2t+11}{-t+r}$$

$$\Rightarrow f(t) = \frac{-1^2t+11}{-t+r} \quad \checkmark$$

$$n^r + \frac{1}{n^r} = \left(n + \frac{1}{n}\right)^r - r\left(n + \frac{1}{n}\right) \Rightarrow f(t) = t^r - rt \quad \checkmark$$

$$g(n) = 0 \Rightarrow n < \frac{1}{\sqrt{r}}$$

$$f(n) = n\sqrt{n} < \begin{cases} 2\sqrt{r} \Rightarrow n < \sqrt{r} \quad \checkmark \\ 1\sqrt{r} \Rightarrow n < 1 \quad \checkmark \end{cases} \quad r=1 \Rightarrow 1 \quad \checkmark$$